THE PROCEEDINGS OF
THE PHYSICAL SOCIETY

VOL. 56, PART 5 1 September 1944 No. 317

SOME NOTES ON SPACE PERCEPTION

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A paper read to the Optical Group; MS. received 24 April 1944

ABSTRACT. Experiments are described illustrating the existence in many people of spatial distortions in binocular vision. These are largely ignored in the observation of everyday scenes when vision is aided by perspective and other factors, but may exercise important influence in special visual tasks. The thesis that these spatial distortions are due to the condition called aniseikonia is reviewed.

§ 1. INTRODUCTION

WHEN we are confronted with a scene consisting of a certain pattern of objects in space our visual mechanism is presented with a task which includes two important steps, namely:—

(i) *Oculo-motor balance*: the focussing and moving of the eyes to maintain binocular fixation on the objects of attention.

(ii) *Perception*: recognition and interpretation of the scene; the fusion of present stimuli from the two eyes with our memory of previous experiences and the sublimation of these into a single percept in the mind.

The study of these processes, and of the numerous anomalies that may arise, constitute the subject of binocular vision. In both the above divisions of the subject we meet with certain anomalies of normal behaviour. Those in the former division are familiar to ophthalmologists and ophthalmists under their two subdivisions of squint and heterophoria, which have been extensively studied, and for which methods, though not as yet very satisfactory methods, of clinical treatment have been developed. The second division, dealing with visual perception and its anomalies, has received far less attention. Although much work has been done on stereoscopy since the time of Wheatstone, the broader aspects of space perception include numerous phenomena for which there is as yet no adequate explanation; and the literature contains many contradictions and unreliable statements.

These difficulties arise because the act of interpreting what we see is not just a physical process subject to the laws of physics and geometry, but is a process involving the neuro-muscular mechanism and, at its highest levels, requiring the participation of the mind. So in the study of space perception we find ourselves often in the realm of psychology; at this stage the physicist and technician hand over the problems to the psychologist and, through lack of collaboration, progress is slow.

The subject of space perception is important, however. When perceptual anomalies are pronounced, they disturb the oculo-motor balance and cause ocular distress. When small, they may not apparently affect vision under ordinary everyday conditions, when precise observation is not required, but they become operative when we are faced with unusual visual tasks. Such tasks often arise in aviation and in the use of certain binocular measuring instruments; under these conditions the observer is denied some of the factors such as parallax, perspective, etc., which normally assist him in forming his visual judgments.

A critical analysis of the status of our existing knowledge of binocular vision, including space perception, is badly needed. All I propose to do on this occasion is to describe briefly one or two simple experimental phenomena that will serve to define the subject and give a slight indication of the type of problem involved. The experiments were carried out at the Northampton Polytechnic.

§ 2. DEFINITIONS

The following terms frequently arise:—

Plane of fixation: the plane containing the centres of projection of the two eyes and the object fixed; when observing an object on the horizon from ordinary altitude, the plane of fixation is sensibly horizontal.

Frontal plane: any vertical plane parallel to the inter-ocular base line.

Median plane: the vertical plane midway between the eyes and perpendicular to the frontal planes.

Size lens: this term has been introduced in connection with the subject of aniseikonia, which is discussed below.

A size lens, or iseikonic lens, is a thick bent lens of zero power. Its surfaces are usually curved in the same sense (figure 1), so that the lens is, in effect, a Galilean

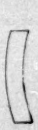

Figure 1. Size or iseikonic lens.

telescope of very small magnifying power. If the back surface has power F_2, and assuming a refractive index of 1·50, a size lens of thickness t mm. magnifies $tF_2/15$ per cent; i.e. 2 per cent for 5 mm. thickness, 4 per cent for 10 mm. thickness, and so on, for a back surface of power -6D.

The magnifying effect of a size lens may be incorporated with an ordinary spectacle lens by modifying the form and thickness of the latter.

§ 3. EFFECT OF SIZE LENS ON STEREOSCOPY

If a person with perfect oculo-motor balance and space perception observes binocularly two isolated objects A and B (figure 2) lying in a horizontal plane through the eyes in a homogeneous field, A and B being placed so that they appear to lie in a frontal plane as indicated, then if a size lens be interposed before, say, the left eye, the object A on the left will appear to move backwards to some

point C; i.e. the plane containing the mental projections of A and B is tilted about a vertical axis through an angle ϕ.

Before the interposition of the size lens the length AB subtends equal angles w at the eyes, the retinal images $a'b'$ and $a''b''$ are equal (assuming equal eyes) and the physiological impressions $\alpha'\beta'$ and $\alpha''\beta''$ conveyed to the brain for fusion may be termed "equal". Out of these, which we may call cortical images, and the acquired experience of the individual, the final single mental percept is built up. The size lens increases the angle w under which the pencils from A and B enter the left eye, $a'b'$ is increased to $b'c'$ and the brain has to fuse the enlarged cortical image $\beta'\gamma'$ with the original $\alpha''\beta''$, as it would if presented with two objects C and B not lying in a frontal plane. The result is that A and B,

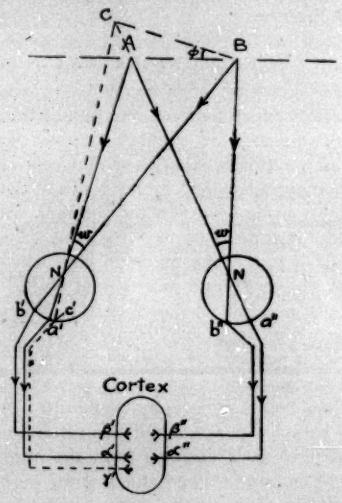

Figure 2.

which appear to be in a frontal plane in unaided vision, now appear to lie in or near the tilted plane CB.

If the fused perceptual image formed in consciousness were immediately *derived* from the retinal images and determined by their relative sizes, we might expect to be able to calculate the amount of stereoscopic effect or tilt ϕ caused by a size lens of given magnification from the geometry of figure 2. Experiment shows, however, that the tilt of field is not generally in accordance with such calculation. Psychological factors, arising probably from the unusual nature of the visual task and variations of attention, appear to falsify the retinal proportionality in forming the final mental image. Moreover, the amount of tilt obtained experimentally depends upon the nature of the scene AB; the tilt obtained when A and B are two isolated objects will not in general agree with the tilt that results when AB is a broken or continuous horizontal line.

However, there *is* some tilt of the field with all observers, provided that A and B are seen in a homogeneous field and the observation is not complicated by the presence of other factors such as perspective, etc.

If the objects A and B lie one *vertically* above the other in a frontal plane, the increase in the vertical dimension of the left retinal image produced by the size lens will not introduce any tilt of the field, but will make the binocular observation of the two objects uncomfortable. It is known that for any two objects to appear at different distances one of them must be seen in diplopia in a direction parallel to the interocular base line when the other is fixated and seen singly; their parallactic angles must differ from one another. The size lens does not produce this condition when A and B are separated vertically; it produces a diplopia of either A or B, depending on which one is being fixated at the given moment, in a *vertical* direction.

§ 4. DISTORTION OF THE FRONTAL PLANE IN STEREOSCOPIC VISION

In a common laboratory experiment for determining the acuity of stereoscopic vision, two equal vertical rods or needles, laterally separated, are viewed binocularly through an aperture so that their supports are invisible and they appear isolated in a homogeneous field. One rod, say the right, remains fixed in position and the other can be moved by the observer, with the help of a cord-and-pulley device, until it appears to lie at the same distance, i.e. in the same frontal plane, as the fixed rod. The precision with which this setting can be made is a measure of the observer's stereoscopic acuity. With a number of observers, it is found that not only do they vary in stereoscopic acuity, but some persistently place the left movable rod further away than the right, whereas others persistently place it nearer. In other words, under the conditions of the experiment, in which the monocular factors of perspective, etc., are absent, the observer's plane of projection is often found to be tilted in one direction or the other about a vertical axis. The amount of this tilt is a function of the lateral separation of the rods and their distance from the observer. With knitting needles separated by 10 cm. and viewed at 4 metres, we have found the tilt to vary, for different observers, from zero to about $\pm 25°$.

In the case of all observers, the interposition before one eye of a size lens magnifying one per cent alters the tilt of the frontal plane by an average amount of 15° or so at 4 metres. This figure is subject to wide variations, even with the same observer.

The experiments have been continued using as objects two equal black spots in a homogeneous white field, nothing else being visible. The spots were 6 mm. in diameter and were viewed at a distance of 2 metres. When arranged with a horizontal separation of 18 mm., both Mr. E. Fincham and I showed a frontal plane tilt in the same direction, though to different extents. When the spots are separated vertically by 18 mm. our tilts are in opposite directions. Mr. Fincham persistently places the upper spot nearer than the lower, whereas I just as persistently place it further away. That is, we have horizontal tilts in the same sense but vertical tilts in opposite senses. With the above dimensions, our vertical tilts are about 12° and 6° respectively.

A size lens placed before one eye modifies the horizontal tilt but has no effect on the vertical tilt.

The above results are obtained from observations carried out in free space. For a given observer, tilts of the field in just the same sense are obtained when similar objects are viewed in a stereoscopic device such as a binocular measuring instrument, though the magnitudes of the effects are not generally the same as in free space.

It thus appears that when the eyes are given the unusual task of judging the relative positions of objects in space by using the stereoscopic sense only, factors of perspective, etc., being absent, one's impression of space is quite likely to be distorted in one direction or another. It would be interesting to investigate whether this distortion is fixed for the individual or whether it can be modified by training or experience.

It should be remarked that the consistency of binocular observations of this kind is not very high; individual settings may have a wide spread, due partly to the difficulty of maintaining the required degree of mental concentration and interest.

§ 5. MONOCULAR ALIGNING OF VERTICALLY SEPARATED OBJECTS

In attempting to gather further information concerning the space distortion described above, it was thought that the vertical tilt might have some relation to the known existence of a difference between the true and the apparent vertical meridians of the retinae. That is to say, if two vertical lines are presented to the eyes, each eye seeing only one line of the pair, then in order that they shall be fused into what appears to be a vertical line in space, one of the two actual object lines has usually to be rotated away from the vertical through an angle of approximately 2 degrees. The meridians of the retinae, which, when projected into space, unite to form a single vertical line projection, are not truly vertical; they are sometimes called the *apparent* vertical meridians of the retinae.

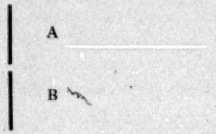

Figure 3.

Accordingly, experiments were carried out on the monocular aligning of vertical lines placed one above the other in the same frontal plane with a gap between their adjacent extremities. Two black lines A and B (A above) of equal width were made by photography on two glass plates. These were mounted vertically, with the aid of a fine plumb bob, as indicated in figure 3. In one series of experiments the lines were 1·0 mm. thick, the gap between their extremities 2·0 mm., their lengths 16 mm., and they were viewed against a white background from a distance of one metre. Line B is fixed and line A can be traversed horizontally right and left by the observer; by using a scale and vernier the position of A can be read to 0·01 mm.

With his head fixed in a head-rest the observer views the lines with one eye and traverses line A laterally until it appears to be directly above line B; its position is read off by a second observer; the mean of ten settings is taken. The settings are repeated for the other eye, the head-rest being shifted sideways so that the second eye occupies the position previously occupied by the first. Figure 4 shows the results obtained for each eye with five observers. Each observer placed the upper line in the position indicated, and then pronounced them to be in vertical alignment.

With the dimensions given above, the sideways displacement of the two lines varies for different observers from zero to about 0·15 mm., which is equivalent to about 30 seconds of arc at 1 metre. For gaps up to 2 mm. this displacement

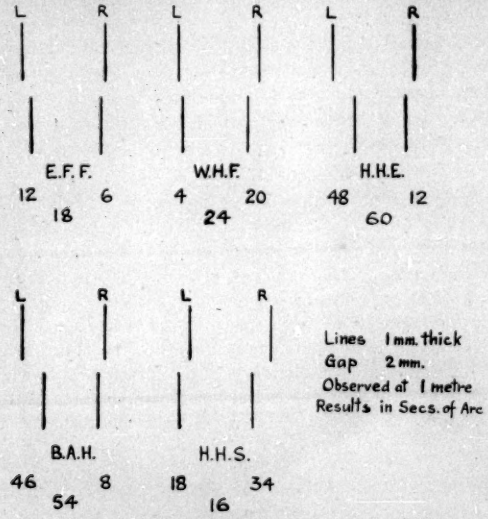

Figure 4. Monocular aligning of separated vertical lines.

is found to be fairly constant for a given observer. This is an interesting phenomenon, but we failed to discover any correlation between these results and those obtained in binocular vision in the previous experiment. It may be significant, however, that the fifth observer, H. H. S., who in this experiment has a bias in a direction opposite to the other observers, also shows marked space distortion in certain binocular experiments.

Similar results are obtained when the vertical lines are replaced by spots of length equal to their width, but we have not investigated the effect of the length of the lines any further.

§ 6. ANISEIKONIA

This is a subject that I approach with some diffidence. It is a remarkable fact that Ames and his colleagues at the Dartmouth Medical School, Hanover,

U.S.A., claim, with the utmost confidence, to measure this condition and to have provided size lenses for its treatment in hundreds of cases in America; and yet we have not succeeded in measuring it nor, indeed, in definitely establishing its existence.

The supposed nature of the condition known as *aniseikonia* (*an*, negative; *isos*, equal; *eikon*, image) may be explained with the aid of figure 2. Objects A and B lying in a frontal plane, the distance AB therefore subtending equal angles at the two eyes, are observed binocularly. If the two eyes are equal, the retinal images $a'b'$ and $a''b''$ of the separation AB are equal. Nevertheless it may happen that the cortical images presented to the brain for fusion are unequal, the inequality arising during the process of transforming and transmitting the retinal images along the optical nerves to the cortex. The inequality may be caused, for example, by lack of symmetry in the distribution of the retinal receptors between the two eyes. However the difference arises, the brain and the mind have to contend with two unequal presentations. In consequence, the individual suffers from asthenopia, due to the strain of fusing the unequal images; and further, his stereoscopic projection is distorted. This space distortion may not be evident in ordinary everyday vision, because we are seldom presented with objects isolated in space; the field usually contains many objects and contours running in various directions and interconnected, in consequence of which we are continuously assisted in our spatial judgments by the monocular factors of perspective, and other factors. But it is claimed that the asthenopia and ocular distress may be serious when the disparity between the two images is only one or two per cent; and further, that these symptoms disappear, in a high proportion of cases, when a size lens of suitable magnifying power is worn before the appropriate eye in order to equalise the two images.

Now there are undoubtedly people who, even after the most careful and skilled examination and treatment of their refraction and oculo-motor imbalance, continue to complain of ocular discomfort. (I doubt whether there are many such people; the complainants are often engaged in special occupations or are generally "nervy".) Also, as we see from the experiments described above, many of us are afflicted with small spatial distortions which may conceivably contribute to the discomfort in ordinary vision and which would certainly affect the stereoscopic vision of a person who is called upon to perform some unusual visual task; e.g. an aviator or the user of certain binocular measuring instruments, in playing certain games of skill, etc. If it can be shown that this condition called aniseikonia is responsible for such cases, then it is clearly of great importance, and aniseikonia treatment should be incorporated in all routine eye examinations.

But first it must be shown that the condition exists; that it can be measured with a precision that is of practical value; that a stable relationship can be established between its degree and the power of optical elements such as size lenses with which it is proposed to correct it. Moreover, before attempting to correct the condition optically, we require information as to whether the condition is of fixed degree in an individual or whether it is capable of alteration by training and experience.

Of these problems, I have time only for a short reference to attempts to detect and measure aniseikonia.

The method that is now proposed by the Dartmouth School is somewhat as follows. Two targets are presented independently to the two eyes (figure 5); the prominent central pattern is the same in each. One target is provided with short lines or dashes disposed on one side of the long radial lines and is seen by, say, the left eye; these dashes are given odd numbers in the diagram. On the other target, presented to the right eye, the short dashes are on the opposite side of the long radial lines (even numbers). The dashes are at an angular distance of 4 degrees from the centre of the target. The arrangement is called an eikonometer. The eyes first observe and fuse the central patterns and then move out, say horizontally, to the short dashes 7 and 8 on the left. If the individual has no aniseikonia in the horizontal direction, the two dashes will be seen exactly opposite one another; if horizontal aniseikonia is present, the dashes will be out of alignment. The degree of aniseikonia thus revealed is to

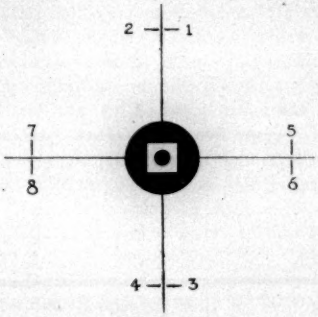

Figure 5.

be measured by interposing before the appropriate eye a size lens or telescopic system of such magnification as to enlarge the retinal image in that eye until the dashes are brought into alignment. The efficacy of the method depends on the assumption that the large central patterns remain fused even although they are now seen in peripheral vision. It is claimed that they do remain fused. If the central patterns are ineffective in holding the eyes in their original state of binocular fixation, then when turned to look at the dashes they are dissociated, and any observed lack of alignment of the dashes is a measure of heterophoria and not of aniseikonia.

With regard to the method of presenting the targets independently to the eyes, a polarization method is adopted by the Dartmouth School. The targets are projected on to a silvered screen by two projectors provided with polaroid screens placed perpendicularly to one another, and the observation is made through a second pair of crossed polaroids. In this way one target only is seen by each eye.

We have tried several methods, including this polarization method, of

presenting the independent targets, but have not succeeded in making any reliable measurements. In early attempts with the polarization method we could not obtain any relative displacement of the dashes with any observer. After a while certain observers reported occasional, but variable, displacements. There were indications, we thought, that the outer vertical edges of the field, and perhaps even the ends of the long radial lines, which are roughly equidistant from the dashes, were being unconsciously fused by the observer, thus locking the eyes for a while in a certain fixation position. Such fixation positions will vary according to the relative distances of the two target edges, or other pairs of marks in the field, from the dashes, and so will falsify the observed displacement of the dashes.

We accordingly made new targets in which the central patterns and all the markings are white on a black background, the radial lines running outwards to the extreme edge and, still using the polarization method, presented these targets in such a way that nothing was visible to the observer but the target marking. With these precautions some observers report varying degrees of displacement of the dashes on some, but not all, occasions.

Thus there is evidence that the central patterns of the targets sometimes remain fused when the eyes are directed towards the dashes, as claimed. But there appear to be other factors preventing reliable measurement.

If with a subject who observes no displacement of the dashes and possessing, therefore, no aniseikonia, a size lens is introduced before one eye, some displacement of the dashes must occur if the central patterns remain fused. We find that this does not always happen, however. After a momentary separation of the dashes as the size lens is introduced, the dashes frequently return to their original positions.

Let us assume that, in spite of the above failure to measure aniseikonia, the condition does in fact exist. Can it then be shown that it is the cause of cases of space distortion? We have seen that both horizontal and vertical tilts of the field are encountered in many individuals. It does not seem possible to establish any connection between magnification in the vertical direction and vertical tilt in the median plane; the latter can only arise as the result of *horizontal* diplopia of one of two vertically disposed objects. It would thus appear that vertical tilt is due to some cause other than aniseikonia. Even with regard to horizontal tilts of the field, the connection with aniseikonia is not too clear. It is true that magnification of one *retinal* image by a size lens produces a horizontal tilt, but there is a basic difference between this and unequal *cortical* images. In the former the angular subtenses of the object are different in the two eyes; in the latter the subtense angles are equal, and it would seem to follow from the known laws of binocular fusion (*vide* Sherrington, 1940) that the unequal cortical images would be projected to the one object in space, in which case no tilt of the field would result.

§ 7. UNEQUAL ILLUMINATION OF STEREOSCOPIC PAIRS

Experience with stereoscopic instruments shows that if two stereograms are presented to the eyes so that they fuse in the usual way into a single projection in depth, then the stereoscopic sense of depth remains when the illumination of

one of the stereograms is reduced to such an extent that it is only dimly discernible. Moreover, as the illumination is reduced, there occurs a lateral distortion of the field. The stereograms of figure 6, for example, are fused into a single spot standing out in front of a distant circle. When the right and left illuminations are equal, the spot appears centrally disposed with respect to the circle, but as the illumination of, say, the left stereogram is reduced, the spot moves laterally towards the left; that is to say, the fused projection assumes to an increasing extent the character of the brighter stereogram.

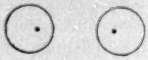

Figure 6.

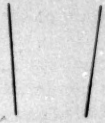

Figure 7.

If the objects presented to the eyes are two nearly vertical lines, as in figure 7, they appear, under equal illuminations, as a single intermediate line with a vertical tilt, the lower end appearing near. If, now, the illumination for the left eye is reduced, the fused line leans over with its upper end towards the right. I find that Verhoeff observed this phenomenon in 1933 and described a method of applying it as a test for stereoscopic vision which he claimed is superior to the usual tests based on depth perception.

§ 8. GENERAL OBSERVATIONS

(*a*) The study of space perception bears directly on the work of the ophthalmologist and ophthalmist. In order to improve and rationalize the present somewhat crude empirical methods of diagnosis and treatment of squint and heterophoria, we need to know more about the causation of these conditions. A more systematic knowledge of the psycho-physical processes of space perception would be of great help in acquiring this knowledge and in dealing with the selection and training of those who are needed for visual work demanding a high standard of binocular vision.

(*b*) In experimenting on visual perception and in the statistical interpretation of the results it would probably be helpful to lean to the view, for which we have support from Sherrington, that the central nervous system on the one hand and the mind on the other play largely independent rôles. On this view the former is just a mechanism; it possesses certain capabilities, some of which are innate in the species and others are acquired during childhood and later experience. It reacts automatically to a given external set of conditions in a manner determined by these characteristics, being incapable of itself of reacting in any other way. In this manner are the subconscious and habitual acts of the body carried out. The mind, associated in some way with the central nervous system, may, and frequently does, intervene, however. According as the attention of the mind is dispersed or is focussed on this or that aspect of the scene, or on some external circumstance, so will the percept and the response of the individual vary. Thus a given scene may evoke different concepts and

different experimental results in the same individual. The seemingly capricious results and " glimpsing " encountered by all experimenters are possibly due in some cases to uncontrollable waverings in attention.

(*c*) Of the several monocular and binocular factors upon which our judgment of the spatial relationship of objects is based, probably the two most important are perspective (monocular) and retinal correspondence or binocular parallax. Subsidiary factors are parallax, size, light and shade, colour, aerial perspective and convergence of the eyes. The interpretation of a given scene will depend upon the relative predominance of perspective features, colour, contrast, etc., in the scene itself and upon the visual mechanism and the mind of the observer at the moment. Those visual characteristics of the observer that are acquired are learnt by the child during its early years of development; the normal sequence of learning is subject to variations as between one individual and another, due to illnesses and other causes. Hence some characteristics may be under- or over-developed, and there may be, in consequence, antagonism between them. This is smoothed out in ordinary vision, when all the factors are in operation, producing a certain resultant spatial percept. But when the individual is confronted with unusual visual tasks, in which some of the factors (perspective in particular) are denied to him, his percept may be distorted.

(*d*) By the adherents of the aniseikonia school, as we may be permitted to call it, this distortion arising in unaided stereoscopic vision is attributed to the one cause, namely aniseikonia or disparity of cortical images. It is admitted that there is evidence in support of the view that a condition of this nature may exist and be a responsible agent. It is true that spatial distortions exist in many people; that distortions in the horizontal direction about a vertical axis are induced by size lenses placed before one eye. But this does not prove that such distortions are caused, or caused only, by unequal *cortical* images. To establish this relation unequivocally, the existence of such image disparity must be established by an independent method. Even so, the observed distortions in the median plane, about a horizontal axis, do not appear to be capable of explanation on the aniseikonia basis. Moreover, more reliable evidence is needed on the stability of the effects of size lenses in stereoscopic vision, alone and in the presence of perspective and other operative factors, and on the variation of such effects with time and experience.

REFERENCES

EMSLEY, H. H., 1936. " The Problem of Unequal Ocular Images " (Ettles Lecture to Institute of Ophthalmic Opticians).

FRENCH, J. W., 1922-23. " Stereoscopy Re-stated." *Trans. Opt. Soc.* **24**, 4.

KOEHLER, W., 1939. *Dynamics in Psychology* (London : Faber and Faber).

LANGLANDS, N. M. S., 1926-27. " Experiments on Binocular Vision. *Trans. Opt. Soc.* **28**, 2.

LIVINGSTON, P. C., 1940. *Opthalmology in Aviation* (*Modern Trends in Ophthalmology*) (London : Butterworth).

MARTIN, L. C., 1926. " Some Visual Factors Affecting the Use of Optical Instruments." *Proc. Opt. Conv.* p. 166.

SHERRINGTON, C. S., 1940. *Man on His Nature* (Cambridge : The University Press).

THOMPSON, S. P., 1877. " Chromatic Aberration of the Eye in Relation to the Perception of Distance." *Phil. Mag.* July.

The numerous papers by A. AMES, G. H. GLIDDON, K. N. OGLE and others on Aniseikonia and related subjects have appeared from 1928 onwards in :

American Medical Association, *Trans. of Section of Ophthalmology*, 1928 ; *Archives of Ophthalmology*, April 1932, Oct. 1938, March and April, 1939 ; *J. Opt. Soc. Amer.* Oct., Nov., Dec., 1932. *Annals, Distinguished Service Foundation of Optometry*, 1932. *Amer. J. Ophthalmology*, Nov. 1935, Sept. 1943.

DISCUSSION

Squadron-Leader W. H. R. JEREMY. In the three-pin test employed in investigating cases of ocular imbalance in the R.A.F. it has been noticed that esophorics tend to end up in front of the pins, exophorics and convergence insufficiencies tend to end up behind. In flying, esophores tend to fly in, exophores and poor convergence cases tend to hold off. Has Mr. Emsley any observations on this, please ?

Mr. N. ARCHBOLD. May I question Mr. Emsley on the subject of aniseikonia and its measurement ? I cannot claim to have read all the literature on this subject, nor, incidentally, have I been convinced by previous publications, but I understand that the diagram exhibited this afternoon is of recent design. If this is correct, am I right in assuming that something occurs with this diagram which has not occurred with others, and which prompted him to modify his remarks and express doubt as to what really is occurring when the diagrams are fused ? I would like to know if there is any peculiarity which has indicated that there is an apparent size difference in the retinal images and also whether, in view of the appreciable angular subtense of the diagram, the central portion is perfectly fused when the markers at the sides are observed.

AUTHOR'S reply. Squadron-Leader Jeremy's observation on the performance of esophoric and exophoric subjects in the three-pin test and in actual flying is interesting. As with other binocular tasks, it seems reasonable to expect that the final mental percept of a given scene might differ as between an individual with normal oculo-motor balance and another individual who has an imbalance and is consequently called upon to exert extra effort to maintain binocular fixation. I always find it difficult, however, to account in detail for the results that are sometimes obtained in experiment or in practice. In the present case, for example, as the observer shifts his gaze back and forth between the fixed pin and the movable pin, why should there be any difference between his judgment of their distances ? Is the result the same if he steadily fixates the central movable pin ?

In work such as stereo-rangefinding, which is a similar binocular task, it has been reported that the ranging is affected by the observer's heterophoric condition. We ourselves have not discovered any such correlation, however, although we have not specially investigated this particular problem.

I am sorry I cannot help the questioner in this matter, but I agree that it is important and would be worth investigating further.

I take it that Squadron-Leader Jeremy, in referring to the use of the three-pin test in investigating cases of ocular imbalance, meant that the observer's stereoscopic sense was under test ?

In reply to Mr. Archbold, the targets illustrated in figure 5 are somewhat different in design from those used earlier by Ames and his colleagues, but the main point is that they are to be observed in a different manner. In the earlier days the subject was directed to fixate the central patterns so that the measuring marks or dashes were seen in peripheral vision. This was criticized ; and now the subject is directed to fixate the measuring marks so that he sees the central patterns in peripheral vision. It is claimed that he nevertheless continues to fuse the central patterns so that aniseikonia is revealed if the measuring marks do not coincide. This claim I discuss in the paper.

COLOURS OF TOTAL RADIATORS EXPRESSED ON THE C.I.E. TRICHROMATIC SYSTEM FOR THE TEMPERATURE RANGE 1500–10–9000° K. ($C_2 = 14384{\cdot}8$)

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MS received 9 *May* 1944.

ABSTRACT. Tables of the colours of Planckian (total or black-body) radiators expressed on the C.I.E. Trichromatic System (1931) are given for the temperature range 1500–10–9000° K., also for 9500–500–11000° K., 13333⅓ °K., 20000° K., 40000° K., and infinite temperature. The colours refer to energy distributions calculated from Planck's formula with a value 14384·8 for the constant C_2. Additional tables are included to facilitate the calculation of the colours if other values of C_2 are used in Planck's formula. The colour differences are given for a change of the constant from 14384·8 to 14350 and from 14384·8 to 14320.

In 1941, tables of the colours of Planckian radiators for the temperature range 1500–10–9000° K. ($C_2 = 14350$) were prepared to meet the requirements of routine test work. The six figures to which they were calculated exceed the requirements for ordinary colour measurements, but tables which can be interpolated to the five-figure accuracy required for some colour-temperature calculations have often been wanted, and it was thought that they might be useful to other workers in this field of colorimetry. These tables were intended to be basic tables from which other tables having different values for the Planck constant C_2 could be obtained by a simple calculation followed by interpolation to tenths.

The colours of the radiators were derived from the Planckian spectral energy distributions using the *Condensed Tables for Colour Computation* (for the equal energy stimulus), by T. Smith (1934). The spectral energy distributions were calculated for the wave-length range 0·38 μ to 0·77 μ from Planck's formula

$$E_{\lambda\theta} = C_1\lambda^{-5}\,[\exp\,(C_2/\lambda\theta) - 1]^{-1},$$

where $E_{\lambda\theta}$ is the amount of energy radiated at the absolute temperature θ between the wave-lengths $\lambda \pm \frac{\delta\lambda}{2}$ (microns) and C_1 and C_2 are constants. Since the energy values were made equal to 100·0000 at 0·56 μ, the value of C_1 was not required. The value 14350 was given to C_2 because a number of calculations had already been made on this basis.

The spectral energy distributions for the temperatures 1500, 1875, 2000, 2125, 2250, 2375, 2500, 3250, 3750, 6000, 6500, 7000, 7500, 8000, 8500, 9000, 9500, 10000, 10500, 11000, 13333⅓, 20000, 40000° K. and for infinite temperature were calculated directly to seven figures, and those for 3000, 3500, 4000, 4250,

4500, 5000 and 5500° K. were obtained from the calculations of the energy distributions for the temperature double the value in the following way:—

For temperature θ,

$$E_{\lambda,\theta} = C_1\lambda^{-5}\,[\exp\,(C_2/\lambda\theta) - 1]^{-1},$$

and for temperature 2θ,

$$E_{\lambda,2\theta} = C_1\lambda^{-5}\,[\exp\,(C_2/2\lambda\theta) - 1]^{-1},$$

so that

$$E_{\lambda,\theta} = E_{\lambda,2\theta}\,[\exp\,(C_2/2\lambda\theta) + 1]^{-1}.$$

If $E_{\lambda,2\theta}$ and $\exp\,(C_2/2\lambda\theta)$ have been calculated for the temperature 2θ, these same values can be used and the relative energy distributions for the temperature θ obtained by single divisions. All the energy distributions were checked by fifth differences and were estimated to be correct to about one part in a million.

The trichromatic coefficients of the colours of the radiators at the previously stated temperatures were calculated, and values for other temperatures then interpolated.

These earlier tables form the basis for those now given with the value 14384·8 for C_2 in Planck's formula (R. T. Birge, 1941). The trichromatic coefficients for the tables with $C_2 = 14384{\cdot}8$ were calculated from those for $C_2 = 14350$ in the following way:—

Two illuminants have the same spectral energy distribution if the factor C_2/θ in Planck's formula remains unaltered. It follows that the trichromatic coefficients calculated (from the $C_2 = 14350$ tables) for the temperatures $\theta_{14350} \times 14350 \times (14384{\cdot}8)^{-1} = 0{\cdot}9975808\,\theta_{14350}$ will correspond to the temperature θ_{14350} on the $C_2 = 14384{\cdot}8$ basis. These calculations of the trichromatic coefficients were made from the 1941 ($C_2 = 14350$) tables for 1500–100–9000° K., and were then differenced to the fifth order. Alterations of one unit in the sixth decimal place were made in a few instances to smooth the values obtained.

Interpolations to tenths were calculated for the X, Y and Z coefficients, and alterations of a unit in the sixth decimal place were made to the Z coefficients where necessary to make the sum of the three coefficients exactly unity. These coefficients are given in table 1.

Additional trichromatic coefficients, which may be in error by 0·00001, are given in table 2. These were derived by additions to the coefficients calculated for $C_2 = 14350$. The additions for 10000° K. and 20000° K. were $\dfrac{14384{\cdot}8 - 14350}{14350 - 14320} = 1{\cdot}16$ times the differences between the coefficients which were calculated for $C_2 = 14350$ and $C_2 = 14320$. The remaining additions were interpolated between those for 9000° K., 10000° K., 20000° K. and infinite temperature.

Several values have been suggested for C_2 over the last fourteen years during which the author has been employed on colorimetry. Two of these values are 14320 and 14350. To convert the present tables to refer to these constants, tables 3–6 may be used. For other values of C_2, linear interpolation may be

(*Continued on page* 328)

Table 1

Temperature in °K. $C_2=14384{\cdot}8$	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
1500	585670	393163	21167
1510	584407	393831	21762
1520	583146	394486	22368
1530	581889	395130	22981
1540	580635	395761	23604
1550	579385	396381	24234
1560	578137	396989	24874
1570	576893	397585	25522
1580	575651	398170	26179
1590	574414	398743	26843
1600	573179	399305	27516
1610	571948	399855	28197
1620	570720	400394	28886
1630	569495	400922	29583
1640	568273	401439	30288
1650	567055	401944	31001
1660	565840	402439	31721
1670	564629	402923	32448
1680	563421	403395	33184
1690	562216	403858	33926
1700	561015	404309	34676
1710	559817	404750	35433
1720	558623	405180	36197
1730	557432	405600	36968
1740	556244	406010	37746
1750	555060	406409	38531
1760	553879	406798	39323
1770	552702	407177	40121
1780	551529	407546	40925
1790	550359	407906	41735
1800	549192	408255	42553
1810	548029	408595	43376
1820	546870	408925	44205
1830	545714	409245	45041
1840	544561	409556	45883
1850	543413	409858	46729
1860	542268	410150	47582
1870	541126	410434	48440
1880	539989	410708	49303
1890	538854	410973	50173
1900	537724	411229	51047
1910	536597	411476	51927
1920	535474	411715	52811
1930	534355	411945	53700
1940	533240	412167	54593

Table 1 (*continued*)

Temperature in °K. C_2=14384·8	Trichromatic coefficients $x\times10^6$	$y\times10^6$	$z\times10^6$
1950	532128	412380	55492
1960	531020	412584	56396
1970	529915	412781	57304
1980	528815	412969	58216
1990	527718	413149	59133
2000	526625	413321	60054
2010	525536	413485	60979
2020	524450	413642	61908
2030	523369	413790	62841
2040	522291	413931	63778
2050	521217	414065	64718
2060	520147	414191	65662
2070	519081	414309	66610
2080	518018	414421	67561
2090	516960	414525	68515
2100	515905	414622	69473
2110	514854	414712	70434
2120	513807	414796	71397
2130	512764	414872	72364
2140	511725	414942	73333
2150	510690	415005	74305
2160	509659	415062	75279
2170	508632	415112	76256
2180	507608	415156	77236
2190	506589	415194	78217
2200	505573	415225	79202
2210	504561	415250	80189
2220	503554	415270	81176
2230	502550	415283	82167
2240	501550	415291	83159
2250	500555	415292	84153
2260	499563	415288	85149
2270	498575	415279	86146
2280	497591	415264	87145
2290	496611	415243	88146
2300	495635	415217	89148
2310	494663	415186	90151
2320	493695	415149	91156
2330	492731	415108	92161
2340	491771	415061	93168
2350	490815	415009	94176
2360	489862	414952	95186
2370	488914	414891	96195
2380	487970	414825	97205
2390	487029	414754	98217

Table 1 (*continued*)

Temperature in °K.	Trichromatic coefficients		
$C_2 = 14384 \cdot 8$	$x \times 10^6$	$y \times 10^6$	$z \times 10^6$
2400	486093	414678	99229
2410	485161	414598	100241
2420	484232	414514	101254
2430	483307	414425	102268
2440	482387	414331	103282
2450	481470	414234	104296
2460	480557	414132	105311
2470	479648	414026	106326
2480	478743	413917	107340
2490	477842	413803	108355
2500	476945	413685	109370
2510	476052	413564	110384
2520	475162	413438	111400
2530	474277	413309	112414
2540	473395	413177	113428
2550	472517	413040	114443
2560	471643	412900	115457
2570	470773	412757	116470
2580	469907	412610	117483
2590	469045	412460	118495
2600	468186	412307	119507
2610	467331	412150	120519
2620	466481	411991	121528
2630	465634	411828	122538
2640	464790	411662	123548
2650	463951	411493	124556
2660	463116	411321	125563
2670	462284	411146	126570
2680	461456	410969	127575
2690	460632	410788	128580
2700	459811	410605	129584
2710	458994	410419	130587
2720	458181	410231	131588
2730	457372	410040	132588
2740	456566	409846	133588
2750	455764	409650	134586
2760	454966	409451	135583
2770	454172	409251	136577
2780	453381	409047	137572
2790	452594	408842	138564
2800	451810	408634	139556
2810	451030	408424	140546
2820	450254	408212	141534
2830	449481	407998	142521
2840	448712	407782	143506

Table 1 (*continued*)

Temperature in °K. $C_2=14384{\cdot}8$	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
2850	447946	407564	144490
2860	447184	407344	145472
2870	446426	407122	146452
2880	445671	406898	147431
2890	444920	406672	148408
2900	444172	406444	149384
2910	443428	406215	150357
2920	442687	405983	151330
2930	441950	405751	152299
2940	441216	405516	153268
2950	440486	405280	154234
2960	439759	405042	155199
2970	439036	404803	156161
2980	438316	404562	157122
2990	437599	404320	158081
3000	436886	404076	159038
3010	436176	403831	159993
3020	435470	403585	160945
3030	434767	403337	161896
3040	434067	403088	162845
3050	433371	402837	163792
3060	432678	402586	164736
3070	431988	402333	165679
3080	431302	402079	166619
3090	430619	401824	167557
3100	429939	401568	168493
3110	429262	401311	169427
3120	428589	401053	170358
3130	427919	400794	171287
3140	427252	400533	172215
3150	426588	400272	173140
3160	425928	400010	174062
3170	425270	399747	174983
3180	424616	399483	175901
3190	423965	399219	176816
3200	423317	398953	177730
3210	422672	398687	178641
3220	422030	398420	179550
3230	421392	398152	180456
3240	420756	397883	181361
3250	420124	397614	182262
3260	419494	397344	183162
3270	418868	397073	184059
3280	418245	396802	184953
3290	417624	396530	185846

Table 1 (*continued*)

Temperature in °K.	Trichromatic coefficients		
C_2=14384·8	$x\times10^6$	$y\times10^6$	$z\times10^6$
3300	417007	396257	186736
3310	416393	395984	187623
3320	415781	395710	188509
3330	415173	395436	189391
3340	414567	395161	190272
3350	413965	394886	191149
3360	413365	394611	192024
3370	412768	394335	192897
3380	412174	394058	193768
3390	411583	393781	194636
3400	410995	393504	195501
3410	410410	393227	196363
3420	409827	392949	197224
3430	409247	392671	198082
3440	408670	392392	198938
3450	408096	392113	199791
3460	407525	391834	200641
3470	406956	391555	201489
3480	406390	391276	202334
3490	405827	390996	203177
3500	405266	390716	204018
3510	404708	390436	204856
3520	404153	390156	205691
3530	403600	389875	206525
3540	403050	389595	207355
3550	402503	389314	208183
3560	401958	389033	209009
3570	401416	388752	209832
3580	400877	388471	210652
3590	400340	388190	211470
3600	399806	387909	212285
3610	399274	387628	213098
3620	398745	387347	213908
3630	398219	387065	214716
3640	397695	386784	215521
3650	397174	386503	216323
3660	396655	386222	217123
3670	396138	385940	217922
3680	395624	385659	218717
3690	395113	385378	219509
3700	394604	385097	220299
3710	394097	384816	221087
3720	393593	384535	221872
3730	393092	384254	222654
3740	392592	383974	223434

Table 1 (*continued*)

Temperature in °K. C_2=14384·8	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
3750	392095	383693	224212
3760	391601	383413	224986
3770	391109	383133	225758
3780	390619	382853	226528
3790	390131	382573	227296
3800	389646	382293	228061
3810	389163	382014	228823
3820	388682	381734	229584
3830	388204	381455	230341
3840	387728	381176	231096
3850	387254	380898	231848
3860	386782	380619	232599
3870	386313	380341	233346
3880	385846	380063	234091
3890	385381	379785	234834
3900	384918	379508	235574
3910	384457	379231	236312
3920	383999	378954	237047
3930	383543	378677	237780
3940	383089	378401	238510
3950	382637	378125	239238
3960	382187	377849	239964
3970	381739	377574	240687
3980	381294	377298	241408
3990	380850	377024	242126
4000	380409	376749	242842
4010	379970	376475	243555
4020	379533	376201	244266
4030	379097	375927	244976
4040	378664	375654	245682
4050	378233	375382	246385
4060	377804	375109	247087
4070	377377	374837	247786
4080	376952	374565	248483
4090	376529	374294	249177
4100	376108	374023	249869
4110	375689	373752	250559
4120	375271	373482	251247
4130	374856	373213	251931
4140	374443	372943	252614
4150	374031	372674	253295
4160	373622	372406	253972
4170	373214	372138	254648
4180	372808	371870	255322
4190	372404	371603	255993

Table 1 (*continued*)

Temperature in °K. C_2=14384·8	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
4200	372002	371336	256662
4210	371602	371070	257328
4220	371204	370804	257992
4230	370807	370538	258655
4240	370413	370273	259314
4250	370020	370008	259972
4260	369629	369744	260627
4270	369240	369480	261280
4280	368852	369217	261931
4290	368467	368954	262579
4300	368083	368692	263225
4310	367701	368430	263869
4320	367321	368169	264510
4330	366942	367908	265150
4340	366565	367648	265787
4350	366190	367388	266422
4360	365817	367129	267054
4370	365445	366870	267685
4380	365075	366611	268314
4390	364707	366353	268940
4400	364340	366096	269564
4410	363975	365839	270186
4420	363612	365583	270805
4430	363250	365327	271423
4440	362890	365072	272038
4450	362531	364817	272652
4460	362174	364562	273264
4470	361819	364308	273873
4480	361465	364055	274480
4490	361113	363802	275085
4500	360763	363550	275687
4510	360414	363298	276288
4520	360067	363047	276886
4530	359721	362796	277483
4540	359377	362546	278077
4550	359035	362296	278669
4560	358694	362047	279259
4570	358354	361798	279848
4580	358016	361550	280434
4590	357680	361302	281018
4600	357345	361055	281600
4610	357012	360808	282180
4620	356680	360562	282758
4630	356349	360317	283334
4640	356020	360072	283908

Table 1 (*continued*)

Temperature in °K. C_2=14384·8	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
4650	355692	359827	284481
4660	355366	359583	285051
4670	355042	359340	285618
4680	354718	359097	286185
4690	354396	358855	286749
4700	354076	358613	287311
4710	353757	358372	287871
4720	353439	358131	288430
4730	353123	357891	288986
4740	352808	357652	289540
4750	352494	357413	290093
4760	352182	357174	290644
4770	351871	356936	291193
4780	351562	356699	291739
4790	351254	356462	292284
4800	350947	356226	292827
4810	350642	355990	293368
4820	350338	355755	293907
4830	350035	355521	294444
4840	349733	355287	294980
4850	349433	355053	295514
4860	349134	354820	296046
4870	348837	354588	296575
4880	348541	354356	297103
4890	348246	354125	297629
4900	347952	353894	298154
4910	347660	353664	298676
4920	347368	353434	299198
4930	347079	353205	299716
4940	346790	352977	300233
4950	346503	352749	300748
4960	346216	352521	301263
4970	345932	352294	301774
4980	345648	352068	302284
4990	345365	351842	302793
5000	345084	351617	303299
5010	344804	351392	303804
5020	344525	351168	304307
5030	344247	350945	304808
5040	343970	350722	305308
5050	343695	350500	305805
5060	343421	350278	306301
5070	343148	350057	306795
5080	342876	349836	307288
5090	342605	349616	307779

Table 1 (*continued*)

Temperature in °K. C_2=14384·8	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
5100	342335	349396	308269
5110	342066	349177	308757
5120	341799	348958	309243
5130	341533	348740	309727
5140	341268	348523	310209
5150	341004	348306	310690
5160	340741	348089	311170
5170	340479	347873	311648
5180	340218	347658	312124
5190	339959	347443	312598
5200	339700	347229	313071
5210	339443	347015	313542
5220	339186	346802	314012
5230	338931	346589	314480
5240	338677	346377	314946
5250	338423	346166	315411
5260	338171	345955	315874
5270	337920	345744	316336
5280	337670	345534	316796
5290	337421	345325	317254
5300	337173	345116	317711
5310	336926	344908	318166
5320	336680	344700	318620
5330	336435	344493	319072
5340	336191	344286	319523
5350	335949	344080	319971
5360	335707	343874	320419
5370	335466	343669	320865
5380	335226	343465	321309
5390	334987	343261	321752
5400	334749	343057	322194
5410	334512	342854	322634
5420	334276	342651	323073
5430	334041	342449	323510
5440	333807	342248	323945
5450	333574	342047	324379
5460	333342	341847	324811
5470	333110	341647	325243
5480	332880	341447	325673
5490	332651	341248	326101
5500	332422	341050	326528
5510	332194	340852	326954
5520	331968	340655	327377
5530	331742	340458	327800
5540	331517	340262	328221

Table 1 (*continued*)

Temperature in °K. $C_2=14384{\cdot}8$	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
5550	331293	340066	328641
5560	331070	339871	329059
5570	330847	339676	329477
5580	330626	339482	329892
5590	330406	339288	330306
5600	330186	339095	330719
5610	329967	338902	331131
5620	329749	338710	331541
5630	329532	338518	331950
5640	329316	338327	332357
5650	329101	338137	332762
5660	328886	337947	333167
5670	328673	337757	333570
5680	328460	337568	333972
5690	328248	337379	334373
5700	328037	337191	334772
5710	327827	337003	335170
5720	327617	336816	335567
5730	327409	336630	335961
5740	327201	336444	336355
5750	326994	336258	336748
5760	326788	336073	337139
5770	326583	335888	337529
5780	326379	335704	337917
5790	326175	335520	338305
5800	325972	335337	338691
5810	325770	335154	339076
5820	325569	334972	339459
5830	325368	334790	339842
5840	325168	334609	340223
5850	324969	334428	340603
5860	324771	334248	340981
5870	324574	334068	341358
5880	324377	333888	341735
5890	324181	333709	342110
5900	323986	333531	342483
5910	323792	333353	342855
5920	323598	333176	343226
5930	323405	332999	343596
5940	323213	332822	343965
5950	323021	332646	344333
5960	322831	332471	344698
5970	322641	332295	345064
5980	322451	332121	345428
5990	322263	331947	345790

Table 1 (*continued*)

Temperature in °K. C_2=14384·8	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
6000	322075	331773	346152
6010	321888	331600	346512
6020	321701	331427	346872
6030	321516	331255	347229
6040	321331	331083	347586
6050	321146	330911	347943
6060	320963	330740	348297
6070	320780	330570	348650
6080	320597	330400	349003
6090	320416	330230	349354
6100	320235	330061	349704
6110	320055	329892	350053
6120	319875	329724	350401
6130	319696	329556	350748
6140	319518	329389	351093
6150	319341	329222	351437
6160	319164	329055	351781
6170	318988	328889	352123
6180	318812	328724	352464
6190	318637	328559	352804
6200	318463	328394	353143
6210	318289	328230	353481
6220	318116	328066	353818
6230	317944	327903	354153
6240	317772	327740	354488
6250	317601	327577	354822
6260	317431	327415	355154
6270	317261	327253	355486
6280	317092	327092	355816
6290	316923	326931	356146
6300	316755	326771	356474
6310	316588	326611	356801
6320	316421	326452	357127
6330	316255	326293	357452
6340	316089	326134	357777
6350	315924	325976	358100
6360	315760	325818	358422
6370	315596	325661	358743
6380	315433	325504	359063
6390	315271	325347	359382
6400	315109	325191	359700
6410	314948	325035	360017
6420	314787	324880	360333
6430	314627	324725	360648
6440	314467	324570	360963

Table 1 (*continued*)

Temperature in °K. $C_2=14384{\cdot}8$	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
6450	314308	324416	361276
6460	314150	324263	361587
6470	313992	324109	361899
6480	313835	323957	362208
6490	313678	323804	362518
6500	313522	323652	362826
6510	313366	323500	363134
6520	313211	323349	363440
6530	313057	323198	363745
6540	312903	323048	364049
6550	312749	322898	364353
6560	312596	322748	364656
6570	312444	322599	364957
6580	312292	322450	365258
6590	312141	322302	365557
6600	311990	322154	365856
6610	311840	322006	366154
6620	311690	321859	366451
6630	311541	321712	366747
6640	311392	321566	367042
6650	311244	321420	367336
6660	311096	321274	367630
6670	310949	321129	367922
6680	310803	320984	368213
6690	310657	320839	368504
6700	310511	320695	368794
6710	310366	320551	369083
6720	310221	320408	369371
6730	310077	320265	369658
6740	309934	320122	369944
6750	309791	319980	370229
6760	309648	319838	370514
6770	309506	319696	370798
6780	309365	319555	371080
6790	309224	319414	371362
6800	309083	319274	371643
6810	308943	319134	371923
6820	308804	318994	372202
6830	308665	318855	372480
6840	308526	318716	372758
6850	308388	318577	373035
6860	308251	318439	373310
6870	308113	318301	373586
6880	307977	318163	373860
6890	307841	318026	374133

Table 1 (*continued*)

Temperature in °K. $C_2=14384{\cdot}8$	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
6900	307705	317889	374406
6910	307570	317752	374678
6920	307435	317616	374949
6930	307300	317481	375219
6940	307166	317345	375489
6950	307033	317210	375757
6960	306900	317075	376025
6970	306767	316941	376292
6980	306635	316807	376558
6990	306503	316673	376824
7000	306372	316540	377088
7010	306241	316407	377352
7020	306111	316275	377614
7030	305981	316142	377877
7040	305851	316011	378138
7050	305722	315879	378399
7060	305593	315748	378659
7070	305465	315617	378918
7080	305337	315487	379176
7090	305210	315357	379433
7100	305083	315227	379690
7110	304957	315098	379945
7120	304831	314968	380201
7130	304705	314840	380455
7140	304580	314711	380709
7150	304455	314583	380962
7160	304331	314455	381214
7170	304207	314328	381465
7180	304084	314200	381716
7190	303961	314074	381965
7200	303838	313947	382215
7210	303716	313821	382463
7220	303594	313695	382711
7230	303472	313570	382958
7240	303351	313444	383205
7250	303231	313320	383449
7260	303110	313195	383695
7270	302990	313071	383939
7280	302871	312947	384182
7290	302752	312823	384425
7300	302633	312700	384667
7310	302515	312577	384908
7320	302397	312454	385149
7330	302279	312332	385389
7340	302162	312210	385628

Table 1 (*continued*)

Temperature in °K. $C_2=14384{\cdot}8$	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
7350	302045	312088	385867
7360	301928	311967	386105
7370	301812	311846	386342
7380	301696	311725	386579
7390	301581	311604	386815
7400	301466	311484	387050
7410	301351	311364	387285
7420	301237	311244	387519
7430	301123	311125	387752
7440	301010	311006	387984
7450	300896	310887	388217
7460	300784	310769	388447
7470	300671	310651	388678
7480	300559	310533	388908
7490	300447	310415	389138
7500	300336	310298	389366
7510	300225	310181	389594
7520	300114	310064	389822
7530	300004	309948	390048
7540	299894	309832	390274
7550	299785	309716	390499
7560	299675	309601	390724
7570	299567	309486	390947
7580	299458	309371	391171
7590	299350	309256	391394
7600	299242	309142	391616
7610	299134	309028	391838
7620	299027	308914	392059
7630	298920	308801	392279
7640	298814	308688	392498
7650	298707	308575	392718
7660	298601	308463	392936
7670	298496	308351	393153
7680	298391	308239	393370
7690	298286	308127	393587
7700	298181	308016	393803
7710	298077	307905	394018
7720	297973	307794	394233
7730	297869	307684	394447
7740	297766	307573	394661
7750	297663	307464	394873
7760	297560	307354	395086
7770	297458	307245	395297
7780	297356	307135	395509
7790	297254	307027	395719

Table 1 (*continued*)

Temperature in °K. $C_2 = 14384{\cdot}8$	Trichromatic coefficients		
	$x \times 10^6$	$y \times 10^6$	$z \times 10^6$
7800	297153	306918	395929
7810	297052	306810	396138
7820	296951	306702	396347
7830	296851	306594	396555
7840	296751	306486	396763
7850	296651	306379	396970
7860	296552	306272	397176
7870	296453	306166	397381
7880	296354	306059	397587
7890	296255	305953	397792
7900	296157	305847	397996
7910	296059	305741	398200
7920	295961	305636	398403
7930	295864	305531	398605
7940	295767	305426	398807
7950	295670	305321	399009
7960	295574	305216	399210
7970	295478	305112	399410
7980	295382	305008	399610
7990	295286	304904	399810
8000	295191	304801	400008
8010	295096	304698	400206
8020	295001	304595	400404
8030	294907	304492	400601
8040	294812	304390	400798
8050	294719	304288	400993
8060	294625	304186	401189
8070	294531	304084	401385
8080	294438	303983	401579
8090	294346	303882	401772
8100	294253	303781	401966
8110	294161	303680	402159
8120	294069	303580	402351
8130	293977	303480	402543
8140	293886	303380	402734
8150	293795	303280	402925
8160	293704	303181	403115
8170	293613	303082	403305
8180	293523	302983	403494
8190	293433	302884	403683
8200	293343	302786	403871
8210	293254	302688	404058
8220	293164	302590	404246
8230	293075	302492	404433
8240	292987	302395	404618

Table 1 (*continued*)

Temperature in °K. $C_2=14384{\cdot}8$	Trichromatic coefficients		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
8250	292898	302298	404804
8260	292810	302201	404989
8270	292722	302104	405174
8280	292635	302007	405358
8290	292547	301911	405542
8300	292460	301815	405725
8310	292373	301719	405908
8320	292286	301624	406090
8330	292200	301528	406272
8340	292113	301433	406454
8350	292027	301338	406635
8360	291942	301244	406814
8370	291856	301149	406995
8380	291771	301055	407174
8390	291686	300961	407353
8400	291601	300867	407532
8410	291516	300773	407711
8420	291432	300680	407888
8430	291348	300587	408065
8440	291264	300494	408242
8450	291181	300401	408418
8460	291098	300309	408593
8470	291015	300216	408769
8480	290932	300124	408944
8490	290849	300033	409118
8500	290767	299941	409292
8510	290685	299850	409465
8520	290603	299758	409639
8530	290522	299668	409810
8540	290440	299577	409983
8550	290359	299486	410155
8560	290278	299396	410326
8570	290198	299306	410496
8580	290117	299216	410667
8590	290037	299126	410837
8600	289957	299037	411006
8610	289877	298948	411175
8620	289798	298859	411343
8630	289718	298770	411512
8640	289639	298681	411680
8650	289560	298593	411847
8660	289482	298505	412013
8670	289403	298417	412180
8680	289325	298329	412346
8690	289247	298241	412512

Table 1 (*continued*)

Temperature in °K.	Trichromatic coefficients		
C_2=14384·8	$x\times10^6$	$y\times10^6$	$z\times10^6$
8700	289169	298154	412677
8710	289091	298067	412842
8720	289014	297980	413006
8730	288937	297893	413170
8740	288860	297806	413334
8750	288783	297720	413497
8760	288707	297634	413659
8770	288631	297548	413821
8780	288554	297462	413984
8790	288479	297376	414145
8800	288403	297291	414306
8810	288328	297206	414466
8820	288252	297121	414627
8830	288177	297036	414787
8840	288102	296952	414946
8850	288028	296868	415104
8860	287953	296783	415264
8870	287879	296700	415421
8880	287805	296616	415579
8890	287732	296532	415736
8900	287658	296449	415893
8910	287585	296366	416049
8920	287512	296283	416205
8930	287439	296200	416361
8940	287366	296118	416516
8950	287293	296035	416672
8960	287221	295953	416826
8970	287149	295871	416980
8980	287077	295789	417134
8990	287005	295708	417287
9000	286934	295626	417440

Table 2

Temperature in °K.	Trichromatic coefficients		
C_2=14384·8	$x\times10^5$	$y\times10^5$	$z\times10^5$
9500	28359	29178	42463
10000	28064	28834	43102
10500	27804	28524	43672
11000	27573	28245	44182
13333⅓	26766	27241	45993
20000	25648	25770	48582
40000	24723	24480	50797
∞	23991	23413	52596

Table 3

Temperature in °K. $C_2=14384{\cdot}8$	Change in the trichromatic coefficients if C_2 is altered from 14384·8 to 14350		
	$x\times10^6$	$y\times10^6$	$z\times10^6$
1500	−462	+245	+217
1600	479	215	264
1700	495	183	312
1800	508	150	358
1900	519	115	404
2000	529	81	448
2100	536	47	489
2200	540	14	526
2300	543	−17	560
2400	544	46	590
2500	543	73	616
2600	540	98	638
2700	536	121	657
2800	531	142	673
2900	524	161	685
3000	517	178	695
3100	509	193	702
3200	500	207	707
3300	492	219	711
3400	483	229	712
3500	474	238	712
3600	464	246	710
3700	455	252	707
3800	446	257	703
3900	436	262	698
4000	426	266	692
4100	417	269	686
4200	407	271	678
4300	398	273	671
4400	389	274	663
4500	380	275	655
4600	372	275	647
4700	365	274	639
4800	356	274	630
4900	348	273	621
5000	340	272	612
5100	332	271	603
5200	324	270	594
5300	317	268	585
5400	310	266	576
5500	304	264	568
5600	297	262	559
5700	290	260	550
5800	284	257	541
5900	278	255	533

Table 3 (*continued*)

Temperature in °K.	Change in the trichromatic coefficients if C_2 is altered from 14384·8 to 14350		
C_2=14384·8	$x\times10^6$	$y\times10^6$	$z\times10^6$
6000	−272	−252	+524
6100	267	250	517
6200	261	247	508
6300	256	244	500
6400	250	242	492
6500	246	239	485
6600	240	236	476
6700	235	234	469
6800	230	231	461
6900	226	228	454
7000	222	225	447
7100	217	223	440
7200	213	220	433
7300	210	218	428
7400	206	215	421
7500	202	212	414
7600	198	209	407
7700	194	207	401
7800	190	205	395
7900	187	203	390
8000	184	200	384
8100	181	197	378
8200	178	195	373
8300	175	193	368
8400	172	191	363
8500	169	188	357
8600	166	186	352
8700	163	184	347
8800	160	181	341
8900	158	179	337
9000	156	177	333

Table 4

Temperature in °K.	Change in the trichromatic coefficients if C_2 is altered from 14384·8 to 14350		
C_2=14384·8	$x\times10^5$	$y\times10^5$	$z\times10^5$
9500	−14	−17	+31
10000	13	16	29
10500	12	15	27
11000	12	14	26
13333⅓	9	11	20
20000	5	7	12
40000	2	3	5
∞	0	0	0

Table 5

Temperature in °K.	Change in the trichromatic coefficients if C_2 is altered from 14384·8 to 14320		
C_2=14384·8	$x\times10^5$	$y\times10^5$	$z\times10^5$
1500	−860	+455	+405
1600	892	399	493
1700	922	340	582
1800	947	278	669
1900	968	213	755
2000	986	149	837
2100	999	86	913
2200	1007	25	982
2300	1011	−33	1044
2400	1013	87	1100
2500	1011	137	1148
2600	1006	184	1190
2700	998	227	1225
2800	988	266	1254
2900	976	301	1277
3000	963	333	1296
3100	948	361	1309
3200	932	387	1319
3300	916	409	1325
3400	899	427	1326
3500	882	444	1326
3600	864	459	1323
3700	847	470	1317
3800	830	480	1310
3900	812	489	1301

Table 5 (*continued*)

Temperature in °K.	Change in the trichromatic coefficients if C_2 is altered from 14384·8 to 14320		
C_2=14384·8	$x\times10^6$	$y\times10^6$	$z\times10^6$
4000	−793	−496	+1289
4100	776	502	1278
4200	758	506	1264
4300	741	509	1250
4400	725	511	1236
4500	708	512	1220
4600	693	512	1205
4700	678	511	1189
4800	662	511	1173
4900	647	509	1156
5000	633	507	1140
5100	618	505	1123
5200	603	503	1106
5300	590	499	1089
5400	577	495	1072
5500	565	492	1057
5600	553	487	1040
5700	540	484	1024
5800	529	479	1008
5900	517	475	992
6000	506	469	975
6100	496	465	961
6200	486	460	946
6300	476	455	931
6400	465	450	915
6500	456	445	901
6600	447	440	887
6700	438	435	873
6800	429	430	859
6900	421	425	846
7000	413	420	833
7100	404	416	820
7200	397	410	807
7300	390	406	796
7400	383	400	783
7500	376	395	771
7600	368	390	758
7700	361	386	747
7800	354	382	736
7900	348	378	726
8000	343	372	715
8100	337	367	704
8200	331	363	694
8300	325	359	684
8400	320	355	675

Table 5 (*continued*)

Temperature in °K.	Change in the trichromatic coefficients if C_2 is altered from 14384·8 to 14320		
C_2=14384·8	$x\times10^6$	$y\times10^6$	$z\times10^6$
8500	−314	−350	+664
8600	309	346	655
8700	304	342	646
8800	299	337	636
8900	294	334	628
9000	290	330	620

Table 6

Temperature in °K.	Change in the trichromatic coefficients if C_2 is altered from 14384·8 to 14320		
C_2=14384·8	$x\times10^5$	$y\times10^5$	$z\times10^5$
9500	−27	−31	+58
10000⅔	25	29	54
10500	23	28	51
11000	22	26	48
13333⅓	16	21	37
20000	9	13	22
40000	4	5	9
∞	0	0	0

(*Continued from page* 306)

employed ; for example, if the alteration to the X coefficient for 2000° K. on changing from C_2=14384·8 to C_2=14350 is −0·000529, the alteration for C_2=14330 is

$$-0{\cdot}000529\times\frac{14384{\cdot}8-14330}{14384{\cdot}8-14350}=-0{\cdot}000833.$$

The X coefficient is therefore 0·526625 − 0·000833 = 0·52579.

The differencing and interpolating required to prepare these tables from the directly calculated values have been carried out by Miss E. D. Brown. The calculations have been carried out as a part of the research programme of the National Physical Laboratory, and this paper is published by permission of the Director of the Laboratory.

REFERENCES

Birge, R. T., 1941. *Rev. Mod. Phys.* **13**, 237.
Smith, T., 1934. *Proc. Phys. Soc.* **46**, 372.

CURRENT PROBLEMS OF VISUAL RESEARCH

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Lecture delivered 24 *May* 1944

§ 1. INTRODUCTION

The problems of visual research discussed in this lecture mainly concern the sensitivity of the retina. Retinal sensitivity is, of course, only one link in the chain of processes connecting the object viewed with the observer's response. In testing retinal sensitivity it is usual to simplify and standardize, as much as possible, the processes which precede the incidence of light on the retina and those which follow the transmission of messages along the optic nerve to the brain. With this object, artificial pupils, fixation points, blinking by metronome and other devices are used to eliminate uncontrolled variations in the stimulation of the retina. Simplification at the other end of the visual process is sought by asking the subject to make only the most elementary judgments about what he sees and by giving him very simple means, such as a "yes-no" key, to record his response.

Results obtained under such simplified conditions are seldom directly applicable to the visual problems of every-day life. These often require experiments in which the complexity and variability of the practical situation are copied rather closely. New visual problems, mostly of a psychological kind, then arise which it would have been equally appropriate to discuss in this lecture, but about which nothing will be said.

§ 2. VARIATIONS IN THE VISUAL THRESHOLD

A very general test of retinal sensitivity is the determination of the threshold increment, or, briefly, the threshold. As a good deal of the discussion which follows is based on measurements of this quantity, a brief description of the test will not be out of place. The eye views a given distribution of brightness—which may be varying with time in a prescribed way—and, at a given moment, a small additional light stimulus is applied at a particular point in the visual field. By repeated trials with different intensities of the additional stimulus, the critical intensity can be determined at which the observer sees the stimulus on fifty per cent of occasions. This threshold increment, or, better, its reciprocal, provides a measure of the sensitivity of a given part of the retina at a given time. It can be determined under a wide range of conditions, and by varying the angular size, exposure time and colour of the test stimulus, the response can be made to depend in different degrees on different mechanisms in the retina.

In a typical determination of the threshold increment by the method now commonly adopted, an S-shaped curve is obtained showing how the chance of seeing the test stimulus varies with its intensity. In the example shown in figure 1 it is apparent that there is a considerable range of stimulus intensities,

of the order of 3 to 1, within which it is a matter of chance whether the stimulus will be seen. This range of indefiniteness is commonly attributed to uncontrollable variations in the sensitivity of the retinal or post-retinal processes in the observer. Some years ago, however, it was suggested in several quarters (Barnes and Czerny, 1932; Stiles, 1932; Brumberg and Vavilov, 1933) that for an eye in its most sensitive state, the threshold is so small that *quantum fluctuations in the stimulus* might be responsible for an appreciable part of the observed scatter.

The threshold, expressed as the number of quanta of radiation entering the eye from the test stimulus, has its smallest value when the eye is fully dark-adapted and when the test stimulus is a very brief flash (0·01 sec. or less) from a point source (10 min. diam. or less) of green light ($\lambda = 510\,\mathrm{m}\mu$) which is viewed by slightly averted vision, so that the image is formed on the parafoveal retina. Under these conditions the threshold has a value of about 50 quanta. Because of the corpuscular nature of radiation, any physical apparatus designed to flash a fixed number, say N, quanta into the eye can do so only on the average, the

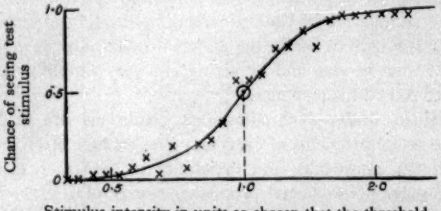

Figure 1. Typical S-shaped curve obtained in a threshold determination.

actual number varying about N with a standard deviation of $\sqrt{N}$. Thus, even if the subject invariably responded when 50 or more quanta entered the eye, a flash of nominally 40 quanta would sometimes contain 51 quanta and would be seen, whereas one of nominally 60 quanta would sometimes contain only 49 quanta and would be missed.

But quantum fluctuations on 50 quanta are too small to explain the whole of the scatter evidenced in the experimental S-shaped curves, and there seemed no satisfactory way of separating quantum from biological fluctuations. Recently a big step forward was made by Hecht, Shlaer and Pirenne (1942), who advanced strong reasons for thinking that quantum fluctuations are the main factor. They made new determinations of the threshold under the optimum conditions indicated above and obtained values ranging from 54 to 148 quanta for seven observers. They estimate that of the light incident on the cornea only about half reaches the retina, the other half being lost by absorption, reflection or scattering in the optic media of the eye. The crux of their argument is now, that of the light reaching the retina, at most 20 per cent is actually absorbed by the visual purple, the light-sensitive substance in the retinal rods which are certainly the end-organs by which the stimulus is seen under the conditions of the threshold

measurements. They arrived at the figure of 20 per cent in the following way. König had measured, many years ago, the absorption of the whole of the visual purple which could be extracted from the human eye. Assuming this quantity of visual purple to be spread as a uniform layer over the retina, the absorption of green light by the layer was calculated to be about 4 per cent. Similar estimates by Wald for the retinae of rabbits and rats had given values of 4 and 13 per cent respectively. These estimates were likely to be too low and were taken as lower limits. To obtain an upper limit a comparison was made between the scotopic visibility curve and the spectral absorption curves of layers of visual purple of different total absorptions, as measured at the wave-length of maximum absorption (500 mμ). The scotopic visibility curve gives the relative energies of different wave-lengths required to produce a given visual effect on the dark-adapted eye, and it is a reasonable assumption that this curve (expressed in quantum units and corrected for light losses in the eye media) will agree with the spectral absorption curve of the layer of visual purple in the retina. For the fairly weakly absorbing layers concerned, the shape of the spectral absorption

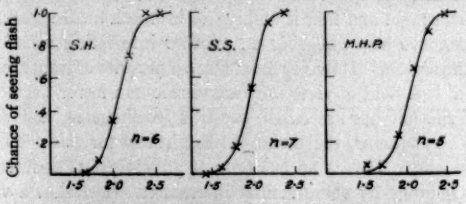

Figure 2. Relation between the chance of seeing the stimulus and the nominal intensity of the stimulus. The plotted points are the experimental values : the curves are calculated from the Poisson distribution assuming the effective number of quanta in the visual act to be 6, 7 and 5 for the respective observers. (From Hecht, Shlaer and Pirenne, 1942.)

curve is not varying very rapidly with the total absorption of the layer, but despite this difficulty Hecht, Shlaer and Pirenne were able to conclude that the scotopic visibility curve is inconsistent with a layer absorbing more than 20 per cent. Thus, applying both corrections, the threshold increment of 54 to 148 quanta measured outside the eye corresponds to an absorption of at most 5 to 14 quanta in the retinal rods, and these latter numbers are the ones on which the quantum fluctuations must be assessed. Assuming that the absorption of n quanta (or more) will always produce a visual response, the S-shaped curve can be calculated from the Poisson probability formula for any value of n. Hecht, Shlaer and Pirenne found that their experimental S-shaped curves fitted the calculated curves for values of n ranging from 5 to 7 (see figure 2). The agreement with the upper limit of 5 to 14 quanta, derived from the absolute value of the threshold, is remarkably close, and these workers draw the conclusion that quantum fluctuations of the stimulus are the main cause of the indefiniteness of the absolute threshold of vision. It may be noted that in reaching this result the careful determinations of the absorbing properties of visual purple by Lythgoe, Wald, Haig and Chase played an important part.

The conclusion applies in the first instance to monochromatic green light (510 mμ), the radiation to which the dark-adapted eye is most sensitive, but it is probably equally valid for other wave-lengths. Such evidence as there is indicates no marked variation in the precision of threshold determinations as the wave-length is changed. For these other colours, although the number of quanta in the threshold flash outside the eye may be many thousands, the number actually absorbed in the rods, and thereby participating in the visual act, is reduced to the same value as for green light by the smaller absorption of the visual purple. It is also possible that when the retina is adapted to brightness levels above zero, the scatter of measurements of the threshold increment is still mainly determined by quantum fluctuations. In the case of rod thresholds there is certainly no marked change in the precision of the measurement as the brightness level is raised. In the case of cone thresholds there is some evidence that the precision is higher than for the rods. At present it is impossible to apply Hecht, Shlaer and Pirenne's argument to cone thresholds as the photochemical substances in the cones have not yet been isolated or even identified. There is no doubt that the question of quantum fluctuations in visual measurements is an important one and that it will figure in much future work.

The S-shaped curve has a bearing on another interesting visual question—the problem of summation. If two or more similar patches of light are sufficiently close together in a dark field of view they are visible at a lower brightness than a single patch. If the patches are small, say 0°·1 in diameter, and contained in an area of about 1° diameter, the threshold brightness for ten patches is about one-tenth that for one. This is an example of physiological summation. The patches assist each other by some retinal interaction which has a very limited radius of action. If the patches are widely separated, however, a kind of summation is to be expected merely as a result of the scatter of the threshold, evidenced in the S-shaped curve. If for a given patch brightness the chance of seeing a single patch is p, the chance P of detecting a group of n patches is the chance of not missing every patch, or $1-(1-p)^n$. This is provided the chance of seeing a given patch is independent of the presence or absence of the other patches. From an experimental S-shaped curve for p, the corresponding curve for P can be calculated, from the formula just given, for any value of n, and the relative thresholds for the single patch and the group can be determined. Meetham and Lambert (1942) discuss this point in their work on the visibilities of groups of light patches seen against a starlit background, and from their S-shaped curve for p they calculate that four patches would reduce the threshold brightness by a factor of 0·85, and that for a very large number of patches the reduction factor would be about 0·6.

Summation of this kind may be called probability summation, to distinguish it from physiological summation, which occurs in the retina or other more peripheral parts of the response mechanism. Its operation does not depend on the cause of the scatter evidenced in the S-shaped curve, which may be either quantum or biological fluctuations or some resultant of the two.

Pirenne (1943) has independently applied a similar idea to binocular summation in the perception of a point flash by the dark-adapted eye. He determined the S-shaped curve for the threshold, using each eye separately and then using

both eyes together. He found that by assuming no physiological summation and only probability summation, the S-shaped curve for binocular vision could be calculated from the curves for the individual eyes, with satisfactory agreement with the experimental results (see figure 3). The binocular threshold in this work was about 80 per cent of the monocular threshold. Binocular summation is a very old problem in physiological optics, but Pirenne's contribution seems to be a most significant one.

The fluctuations of the visual threshold discussed above are the irreducible variations which occur in the comparative short period of a set of measurements during which the physical and physiological conditions are held as steady as possible. They are what may be called, perhaps, "fluctuations of the instantaneous threshold". The visual threshold is also subject to many true physiological and biological variations, and these, especially where they affect the

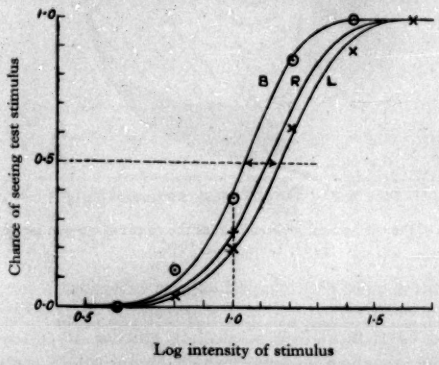

Figure 3. Comparison of the chance of seeing the stimulus with the right eye only (+), the left eye only (×), and the two eyes together (⊙). The curve B for the two eyes is calculated from the curves R and L fitted to the experimental points for the separate eyes, on the assumption of probability summation only. (From Pirenne, 1943.)

threshold of the dark-adapted eye, have attracted a great deal of attention in recent years. A few of these may be illustrated by the results of tests made by a clinical technique recently developed by Livingston (1944) for the study of the fully dark-adapted retina. Livingston uses an extremely weak point-source of light of fixed candle-power at a certain distance from the eye and determines the areas of the subject's field of view within which the light can be seen as it is moved in from, or out to, the periphery.

[A number of charts of the dark-adapted visual field were then shown. These charts bring out the following points:—

(i) The wide differences between individuals with good and poor night vision.

(ii) The temporary improvement produced by a single large dose of vitamin A in the case of a person whose visual field has become

"contracted" by living on a vitamin A deficient diet for some months: the permanent restoration of his visual field by regular but smaller doses of vitamin A.

(iii) The contraction of the visual field as a result of oxygen-lack (anoxia) after 15 min. at an altitude of 17,000 ft. (figure 4).]

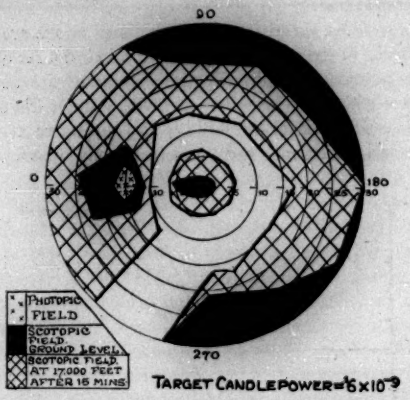

Figure 4. Contraction of the dark-adapted visual field as the result of oxygen lack at high altitude. (From Livingston, 1944.)

No survey of these more physiological aspects of vision is attempted here, but it may be mentioned that spells of deep-breathing or light exercise, weak stimulation by red light, the taking of alcohol, glucose or certain drugs, and several other factors, all affect, or have been claimed to affect, the sensitivity of the dark-adapted eye.

§ 3. VISIBILITY CURVES UNDER DIFFERENT CONDITIONS

Turning back to the more physical aspects of retinal sensitivity, something will now be said of the visibility curve of the eye under different conditions. Originally by *the visibility curve* was meant a curve showing the relative energies of different wave-lengths required to produce a given brightness, as measured in a photometric matching field. In later work, the term has been applied, in a broader sense, to curves showing the relative energies of different wave-lengths required to produce any particular visual effect under prescribed conditions. Thus the threshold increments for different colours, if measured in energy units, determine a visibility curve. It would have been appropriate for Hecht, Shlaer and Pirenne to use the threshold visibility curve for the dark-adapted parafovea in their comparison with the spectral absorption curves of layers of visual purple. In fact, they used a visibility curve determined (many years earlier) by Hecht and Williams (1922) by brightness matching in a 22° diameter field at a brightness about 2·7 times the threshold. The criticism might be made that the concentration of the visual purple must have been lower

in the brightness-matching experiments than it was in the threshold experiments. However, in the region of the spectrum important for their comparison the threshold visibility curves obtained by other workers agree pretty well with their brightness-matching curve.

Accepting the view that the absorption by the visual purple in the fully dark-adapted retina is at most 20 per cent for green light, the slight modifications in the form of the visibility curve resulting from the reduction of the concentration of visual purple at higher brightness levels would be hardly large enough to detect experimentally. In fact, the visibility curve of any particular region

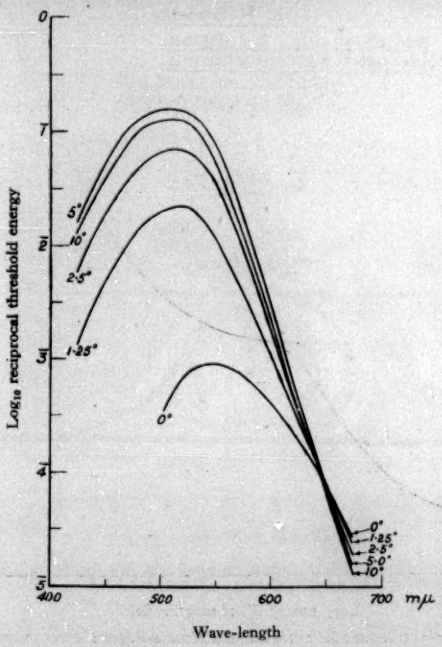

Figure 5. Threshold visibility curves for the fovea and for parafoveal points at various angular distances from the fovea. (From Abney and Watson, 1916.)

of the dark-adapted retina can probably be regarded as a unique curve, provided that only the retinal rods participate in the visual response used in determining the curve. This last condition tends to break down in the red end of the spectrum. For example, Abney and Watson's threshold curves (figure 5) show that the characteristic rod maximum at about 510 mμ is well developed for all retinal points more than 1¼° from the fovea (Abney and Watson, 1916). In the red, however, the parafoveal curves converge on the foveal (0°) curve, which is a pure cone curve. This suggests that in the red the observed parafoveal threshold may be determined wholly or in part by the cones.

A similar difficulty arises in the brightness-matching method. It is only recently that parafoveal visibility curves have been determined by this method, using a sufficiently small matching field for the retinal properties within it to be fairly constant.

In this work of Walters and Wright (1943) measurements were made from near threshold up to quite high brightnesses of the matching field, and the gradual change in the form of the visibility curve from a typical rod curve with maximum at about 510 mμ to a typical cone curve with maximum at about 560 mμ was established. At the lowest brightnesses used, the form of the visibility curve

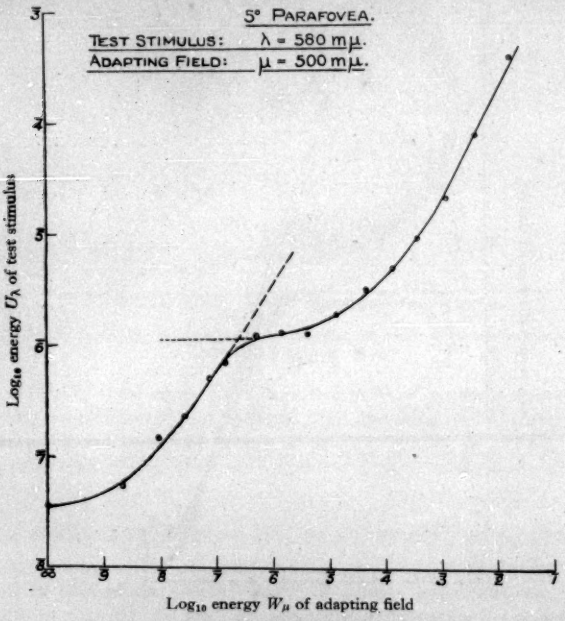
5° PARAFOVEA.
TEST STIMULUS: λ = 580 mμ.
ADAPTING FIELD: μ = 500 mμ.
Log₁₀ energy U_λ of test stimulus
Log₁₀ energy W_μ of adapting field

Figure 6. 5° parafoveal threshold for a test stimulus of wave-length 580 mμ and an adapting field of wave-length 500 mμ. (From Stiles, 1939.)

in the red end was still changing, indicating that the cones were still having some effect.

It is of interest to try and interpret the changes in the parafoveal visibility curve, using the picture of the rod-cone transition which is presented by measurements of the threshold. This picture has emerged from many investigations, but the early work of Hecht and his co-workers should be mentioned. The change in the parafoveal threshold as the brightness level is raised follows a curve such as that shown in figure 6, which refers to a green adapting-brightness and a yellow test-stimulus. There is no reasonable doubt about the meaning of this curve. The lower part represents the threshold of the

rod mechanism and would be continued at still higher brightnesses on the course shown by the broken line if the cone mechanism were absent. Similarly, the upper part represents the threshold of the cones. In measurements of this type there is a tendency near the intersection of the two component curves for the observed threshold to be rather smaller than either of the component thresholds—that is to say, there is a certain amount of summation. Possibly this summation is no more than the probability summation between effectively independent mechanism which was mentioned earlier. Whatever the cause, however, it is certain that in amount it falls short of a complete linear summation. If $1/U_{\lambda s}$ and $1/U_{\lambda p}$ are the reciprocal thresholds for rods and cones at any

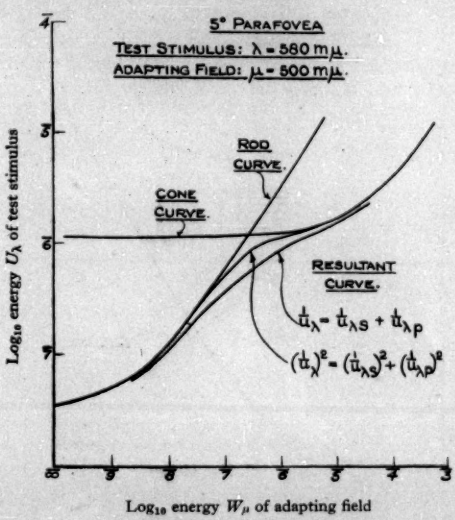
5° PARAFOVEA
TEST STIMULUS: λ = 580 mμ.
ADAPTING FIELD: μ = 500 mμ.
ROD CURVE.
CONE CURVE.
RESULTANT CURVE.
$\frac{1}{U_\lambda} = \frac{1}{U_{\lambda S}} + \frac{1}{U_{\lambda P}}$
$\left(\frac{1}{U_\lambda}\right)^2 = \left(\frac{1}{U_{\lambda S}}\right)^2 + \left(\frac{1}{U_{\lambda P}}\right)^2$
Log₁₀ energy U_λ of test stimulus
Log₁₀ energy W_μ of adapting field

Figure 7. Resultant of rod and cone threshold curves for the case shown in figure 6, assuming different degrees of summation.

brightness level, the observed reciprocal threshold $1/U_\lambda$ may be expressed in the empirical form

$$\left(\frac{1}{U_\lambda}\right)^n = \left(\frac{1}{U_{\lambda s}}\right)^n + \left(\frac{1}{U_{\lambda p}}\right)^n, \qquad \ldots\ldots(1)$$

where $n=1$ would correspond to linear summation. The resultant curves obtained from the rod and cone component curves by assuming $n=1$ and $n=2$ are shown in figure 7. By examination of many experimental curves of this type it is found that $n=1$ gives too gradual a transition between the rod and cone curves: $n=2$ suits better and, as it is has some theoretical reasons in its favour, it may be used tentatively.

The modification of the experimental threshold curve when the wave-lengths of test light and adapting field are changed in turn has been determined

(Stiles, 1939). Experiment shows that if the wave-length of the test stimulus is changed, say from 580 to 500 mμ, the rod component curve is moved bodily down to lower energy thresholds because the rods are more sensitive to light of wave-length 500 mμ. The cones, however, are less sensitive to this wave-length and the cone curve moves up. Change in the wave-length of the adapting field produces similar bodily shifts of the component curves, but parallel to the horizontal axis. Thus, by ringing the changes on test stimulus and adapting wave-lengths, the experimental curve can be moved about the diagram, with considerable relative motion between the rod and cone sections. The extreme

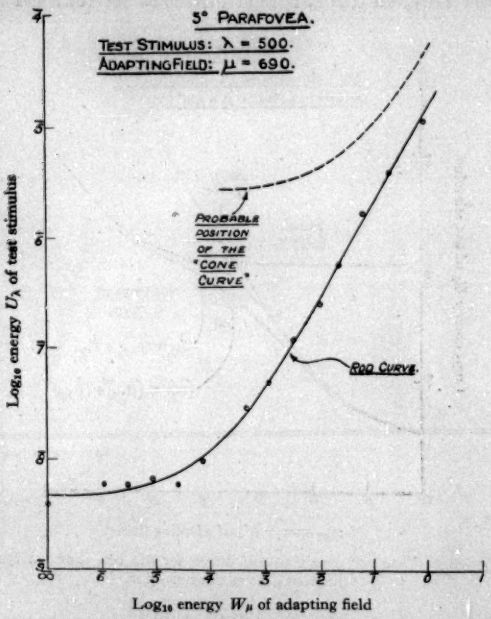
5° PARAFOVEA.
TEST STIMULUS: λ = 500.
ADAPTING FIELD: μ = 690.
PROBABLE POSITION OF THE "CONE CURVE"
ROD CURVE.
Log10 energy Uλ of test stimulus
Log10 energy Wμ of adapting field

Figure 8. 5° parafoveal threshold for a test stimulus of wave-length 500 mμ and an adapting field of wave-length 690 mμ. (From Stiles, 1939).

case shown in figure 8 is of interest. Here the cone curve lies completely to the left of the rod curve and all the observed thresholds are rod thresholds. These experimental findings are summed up as follows:—

If the relative visibility curve of the rods is s_λ, the rod threshold $U_{\lambda s}$ for a test stimulus of wave-length λ and an adapting field of wave-length μ and energy intensity W_μ is given by $s_\lambda U_{\lambda s} = F_s(W_\mu s_\mu)$, where $F_s(x)$ is a fixed function determined by the shape of the rod section of the experimental curves. Similarly for the cone threshold $U_{\lambda p}$ we have $p_\lambda U_{\lambda p} = F_p(W_\mu p_\mu)$, where p_λ is the visibility curve of the cones and $F_p(x)$ is a fixed function determined by the shape of the cone curve.

The condition (1) for the resultant threshold U_λ may thus be written

$$1 = \left[\frac{s_\lambda U_\lambda}{F_s(s_\mu W_\mu)}\right]^2 + \left[\frac{p_\lambda U_\lambda}{F_p(p_\mu W_\mu)}\right]^2,$$

or, generalizing for test stimuli and adapting fields which are not monochromatic,

$$1 = \left[\frac{\int s_\lambda U_\lambda d\lambda}{F_s(\int s_\mu W_\mu d\mu)}\right]^2 + \left[\frac{\int p_\lambda U_\lambda d\lambda}{F_p(\int p_\mu W_\mu d\mu)}\right]^2,$$

or

$$1 = \left[\frac{\delta\sigma}{F_s(\sigma)}\right]^2 + \left[\frac{\delta\pi}{F_p(\pi)}\right]^2, \qquad \ldots\ldots(2)$$

where $\delta\sigma$, σ, $\delta\pi$, π are the scotopic and photopic values (or co-ordinates) of the test stimulus and adapting field.

Turn now to the parafoveal brightness matching field, and suppose one-half has wave-length λ and intensity W_λ. Its scotopic and photopic values are

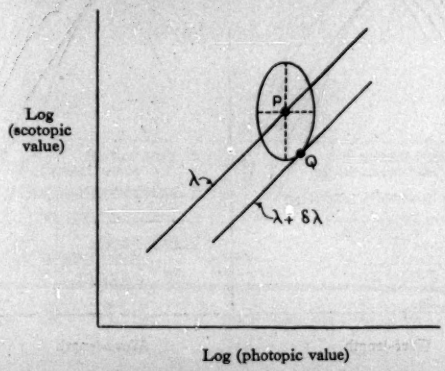

Figure 9. Diagram illustrating the derivation of the relative energies of neighbouring spectrum colours which just match in brightness.

$\sigma = W_\lambda s_\lambda$ and $\pi = W_\lambda p_\lambda$, and it can be represented in the diagram of figure 9 as the point P. In this diagram the points on any line at 45° represent lights of the same spectral composition (strictly of the same ratio of scotopic to photopic value) but of different intensities. By using the threshold condition (2) a small "ellipse" can be drawn round P within which lie the representative points of all lights which would be indistinguishable from P if they occupied the other half of the matching field. The two lines inclined at 45° to the axes which touch this ellipse correspond to lights whose wave-lengths are respectively slightly above and below λ, and for each of them the point of contact Q defines an intensity at which the two halves of the matching field will be indistinguishable and therefore in brightness match.

It is clear that by repeated application of this procedure the energies of a series of wave-lengths in the spectrum can be determined, each of which is in brightness match with its neighbours. In other words, a step-by-step visibility

curve can be calculated for any energy W_λ of the initial wave-length. The basic principle of this method is due to Helmholtz (1896).

Figure 10 shows the results of such a calculation, assuming for p_λ, the C.I.E. visibility curve, for s_λ a mean scotopic visibility curve (Stiles and Smith, 1944) judged to be the closest approach to the true rod curve, and for $F_s(x)$ and $F_p(x)$ functions derived in recent measurements of the threshold increment. In the same figure are plotted Walters and Wright's experimental visibility curves for a 3° parafoveal point. As the brightness level is raised, both the observed and theoretical curves show initially a rise in the red and later a shift of the maximum towards the red, and the intensities at which these changes occur are in fair agreement. It should be observed, however, that the step-by-step method was not used by Walters and Wright: they used a fixed red comparison field. At high brightnesses the step-by-step and the fixed comparison-field

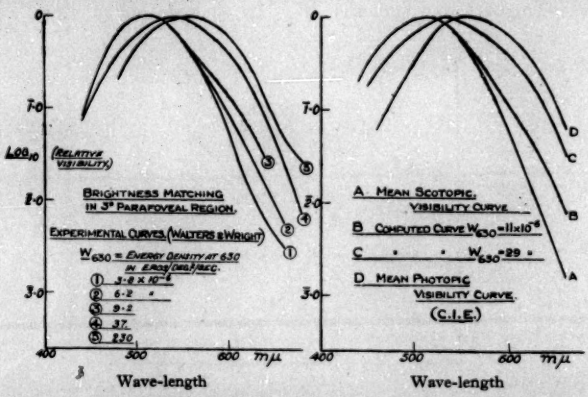

Figure 10. Visibility curves determined by brightness matching with a small field at 3° in the parafovea : (*a*) experimental curves (from Walters and Wright, 1943) ; (*b*) curves derived by the Helmholtz method.

visibility curves are substantially the same, but this may not be so in the transitional region. Nevertheless, this tentative application of the Helmholtz method suggests that it may prove of use in interpreting on a common basis the threshold and brightness matching results.

The foveal curves obtained by Walters and Wright show comparatively little change as the brightness level is varied. This would be consistent with the conception of the cones as a single mechanism with a unique spectral sensitivity curve. But trichomatic theory requires that the cones shall in some way exhibit three spectral sensitivity curves, r_λ, g_λ, b_λ, and the cone visibility curve is usually regarded as a linear form,

$$V_\lambda \text{ (or } p_\lambda) = L_r r_\lambda + L_g g_\lambda + L_b b_\lambda$$

in these quantities. Constancy of the visibility curve then implies that the luminosity coefficients L_r, L_g and L_b do not alter in relative value as the

brightness level changes. This scheme of ideas works fairly well in photometry and colorimetry. In trying to understand the mechanism of retinal response, however, the Helmholtz method of deriving a step-by-step visibility curve is very attractive, and it can also be applied in this case. Here, instead of two mechanisms, rods and cones, there are three mechanisms to consider, the three kinds of cone. Recent measurements of foveal thresholds provide the requisite raw material for the calculation. It has been found that for certain pairs of wave-lengths of test stimulus and adapting field the foveal threshold curve shows changes of law very similar to those corresponding to the rod-cone transition in parafoveal vision. Figure 11 shows an example for the case $\lambda = 480$, $\mu = 540$. It is highly probable that the different sections of such curves are the threshold curves of different cone mechanisms. By observing the way these component curves move in the diagram when the wave-lengths of the test stimulus and

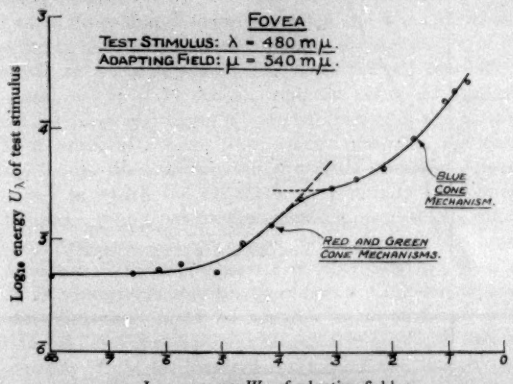
FOVEA
TEST STIMULUS: λ = 480 mμ.
ADAPTING FIELD: μ = 540 mμ.
BLUE CONE MECHANISM.
RED AND GREEN CONE MECHANISMS.
Log₁₀ energy U_λ of test stimulus
Log₁₀ energy W_μ of adapting field

Figure 11. Foveal threshold for a test stimulus of wave-length 480 mμ and an adapting field of wave-length 540 mμ. (From Stiles, 1939.)

adapting field are changed, a rough determination of the spectral sensitivity curves has been made. These are sufficiently near one of the sets obtainable from colour-matching data to justify their identification, as referring to the same three mechanisms. The threshold condition for a brightness-matching field in the fovea then takes the form

$$1 = \left[\frac{\delta x}{F_r(x)}\right]^2 + \left[\frac{\delta y}{F_g(y)}\right]^2 + \left[\frac{\delta z}{F_b(z)}\right]^2, \qquad \ldots\ldots(3)$$

where
$$\begin{cases} \delta x = \int r_\lambda U_\lambda d\lambda = \int r_\lambda (W_\lambda - W_\lambda')d\lambda = \int r_\lambda W_\lambda d\lambda - \int r_\lambda W_\lambda' d\lambda, \\ x = \int r_\lambda W_\lambda d\lambda. \end{cases}$$

$$\begin{cases} \delta y, & \delta z, \\ y, & z, \end{cases} \quad \text{are similarly defined,}$$

and $W_\lambda d\lambda$ and $W_\lambda' d\lambda$ are the energy distributions in the two halves of the field. Here $F_r(x)$, $F_g(x)$, $F_b(x)$ are fixed empirical functions. The derivation of the

step-by-step visibility curve by the Helmholtz method follows the same lines as for the rod and cone case except that the representative points of various lights have now to be plotted in a three-dimensional space. Calculations on these lines give the plotted points in figure 12. They represent the derived visibility curves (*a*) for very high field brightness and (*b*) for a brightness at the level normally used in photometry. The agreement with the C.I.E. visibility curve is very promising. It remains for future work to show whether this line of approach can account for other aspects of the brightness-matching problem—in particular, whether it is consistent with the approximate additivity of brightness under ordinary conditions of photometry.

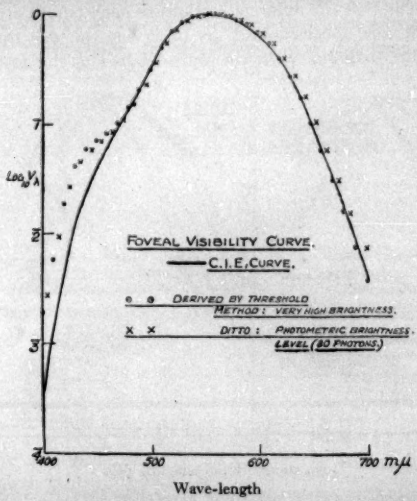

Figure 12. Visibility curve by brightness matching in the fovea: the curve is the C.I.E. standard visibility curve and the plotted points are values derived by the Helmholtz method.

Referring back for a moment to the threshold condition at the fovea, relation (3), it may be noted that at sufficiently high brightnesses this reduces to the form

$$1 = \frac{1}{f_r^2}\left(\frac{\delta x}{x}\right)^2 + \frac{1}{f_g^2}\left(\frac{\delta y}{y}\right)^2 + \frac{1}{f_b^2}\left(\frac{\delta z}{z}\right)^2, \qquad \ldots\ldots(3a)$$

where the constants f_r, f_g, f_b are the limiting values of the Fechner fractions of the three cone mechanisms at high brightnesses. Experimentally f_r, f_g and f_b come out, very roughly, in the ratio 0·8 : 1 : 4·5, so that the blue mechanism has a much higher Fechner fraction than the red or green (Stiles, 1939). In the Helmholtz method of considering the visibility curve, a higher Fechner fraction of the blue mechanism corresponds to the lower luminosity coefficient of the blue response in the more usual treatment.

The colour perceptions in the parafoveal retina are not radically different from those in the fovea, and it must be assumed that there too three cone

mechanisms are operative. The discussion of the parafoveal visibility curve should, therefore, have been based on a four-dimensional threshold condition:

$$1 = \left[\frac{\delta\sigma}{F_s(\sigma)}\right]^2 + \left[\frac{\delta x}{F_r(x)}\right]^2 + \left[\frac{\delta y}{F_g(y)}\right]^2 + \left[\frac{\delta z}{F_b(z)}\right]^2. \quad \ldots\ldots(4)$$

It is not difficult to see, however, that the resulting visibility curves would not be materially different from those obtained on the simpler view. But the form of the result raises a difficult question in our ideas of the visual mechanism. At the fovea, the two halves of the field will match if x, y and z are very small, that is, if three relations of the form $\int r_\lambda W_\lambda d\lambda = \int r_\lambda W_\lambda' d\lambda$ are satisfied. In the parafovea it appears that four such relations would have to be satisfied, which apparently contradicts the main tenet of the trichomatic theory. Guild formulated this difficulty very sharply in the Discussion on Vision, 1932. There are extenuating circumstances. The rods have a high limiting Fechner fraction, perhaps 15 to 50 times that of the green cones, and when δx, δy and δz, for any pair of real lights, are all zero the difference in the stimulation of the rods in the worst case will be no more than a few times the rod threshold, i.e. $\delta\sigma/F_s(\sigma)$ will be a small integer. On this showing the threshold condition (4) would indicate that for most real lights a complete match in the parafovea could be obtained by a mixture of three independent primaries, but that in specially chosen "bad" cases there would still be a supra-threshold difference whatever the proportions in which the primaries were mixed. Presumably this outstanding difference would not be describable in general as a difference of hue, saturation or brightness. Psychologists, in particular Katz, have emphasized that hue, saturation and brightness are not the only modes of appearance of colours. It is conceivable that another modality, for example a "filminess-solidity" differentiation, might be manifest in parafoveal vision, and that to equate parafoveal fields in this respect as well as in hue, saturation and brightness a fourth variable would be required. However this may be, the immediate requirement for an attack on this interesting visual problem is a thorough investigation of colour-matching in the parafovea.

Before leaving the visibility curve, brief reference must be made to a series of investigations by photometrists on the precise form of the curve under the various normal conditions of photometric matching. This work, recently summarized by Buckley (1941), was directed primarily to the practical problem of evaluating the light from the modern gaseous discharge lamps, but it has yielded much new evidence on the effects of field size, brightness level and other measuring conditions, which a satisfactory theory of the visibility curve will have to explain. One novel suggestion of Preston's (1939), later confirmed by Dresler (1940), is that the visibility curve undergoes a slight seasonal variation, the eye in summer being about 5 per cent more sensitive to green (546 mμ) compared with yellow (589 mμ) than it is in winter. This might be connected with seasonal change in the body's vitamin-A content (Preston, 1939).

§4. THE RETINA IN A STATE OF CHANGE

Much more complicated situations arise when the retina is in a state of change. Measurements of the threshold increment again provide a powerful

method of attack. They have been used, for example, to follow the recovery of sensitivity after extinguishing a high brightness to which the retina has become adapted. In the parafovea, the recovery curves for colours other than red show the familiar division into an initial cone section and a later rod section. In the fovea a simple cone recovery curve is observed. The early photochemical

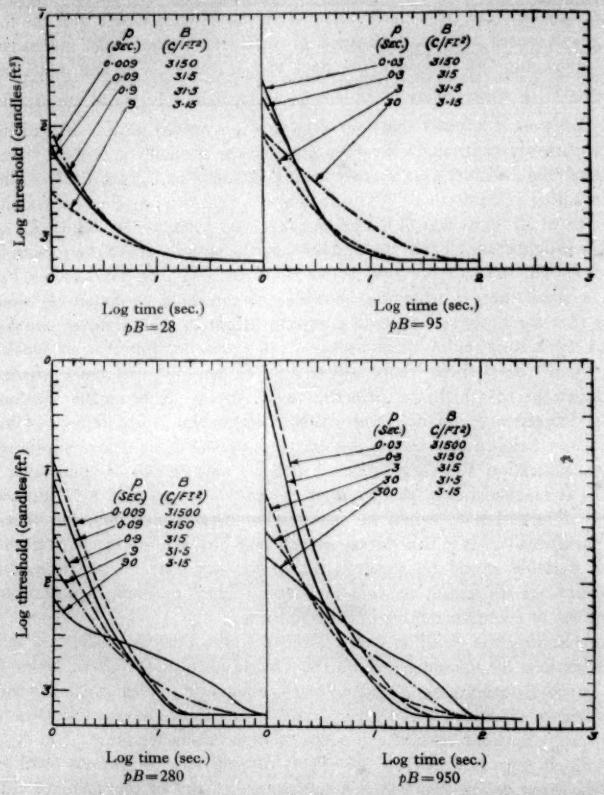

Figure 13. Curves showing the recovery of the foveal threshold after exposure to adapting fields of different durations (p) and brightnesses (B). In each group of curves the product pB is constant. Time is measured from the instant when the adapting field is extinguished. (From Crawford, 1940, reproduced by permission of the Council of the Royal Society.)

theories of the recovery process were based on the idea of a single photochemical substance dissociated to a given degree by the adapting light and then re-forming in the dark in a chemically closed system. On such a view, if the retina had recovered to a given level, the subsequent recovery curve would be the same whatever had been the initial adapting brightness. Experiment shows this is

not the case, and that threshold recovery curves form a family depending on at least two independent parameters. This point is brought out very clearly in the recovery curves recently determined by Crawford (1940), using as adapting light flashes of various brightnesses and durations. The foveal recovery curves in each of the groups shown in figure 13 were all obtained with the same product of brightness and duration. It will be seen that the curves intersect and then diverge, before finally converging to the final threshold. This dependence of the recovery curves on at least two independent parameters occurs for both the rod and the cone recovery processes, and cannot be explained by differences in the rates of recovery of the different rod and cone mechanism. Rather is it a problem in the photochemistry of each mechanism. We may suggest complications such as chain reactions, diffusing away of the products of dissociation, limited capacity of a reservoir of photochemical material, but so far no great progress has been made. The study of the effects of changes in the physiological environment, such as oxygen lack, may prove helpful here.

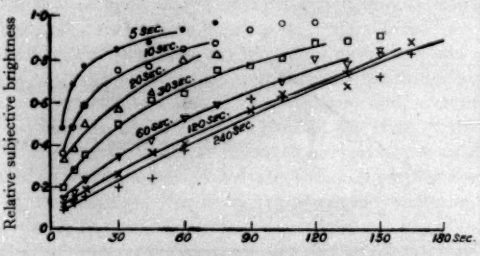

Figure 14. Recovery of the subjective brightness of a test surface after an adapting field of brightness 70 c./sq. ft. approx. exposed for various periods. (From Schouten, 1939.)

Retinal recovery can also be followed by measuring subjective brightness by the method of binocular matching. In this method, as used by Schouten (1939), the left eye views a fixed comparison light patch, while the right eye views a test patch whose intensity can be varied to give a brightness match. After adapting the right eye *only* to a high brightness, the intensity of the test patch must be increased to keep it in match, and in this way a recovery curve can be determined. Figure 14 shows recovery curves after exposure to 70 c./sq. ft. for different periods. The quantity plotted is the relative subjective brightness, which is defined as inversely proportional to the intensity of the test patch. Again, differences in the shape of the recovery curves show that at least two independent parameters are at work. The curves of figure 14 are for foveal vision of the test patch. In what follows we shall be concerned exclusively with foveal or cone properties, and it will be unnecessary to mention it again.

The precise connection between recovery curves obtained by the two methods is not yet clear. The comparison in figure 15 shows that the threshold has fully recovered in 100 sec. or less, while the subjective brightness goes on

increasing for 200 sec. or more. It might have been expected that the threshold measurements would provide a more sensitive indication of the final stages of the recovery process, but the reverse seems to be the case.

The threshold measurements in figure 15 extend to 0·2 sec. after the extinction of the adapting field, and there is no indication that the curves would not extend back, without discontinuity, to the threshold value obtained while the adapting field is still on. This behaviour of the threshold suggests that it will be extremely difficult to determine the state of the retina while the adapting field is actually on, by measurements of subjective brightness made a second or more after it has been switched off. What part, if any, is played by a positive after-image of the adapting field in determining the threshold and subjective brightness-recovery curves is not yet known. Reference should be made here to a comprehensive investigation by Craik (1940) of the effect of adaptation on subjective brightness.

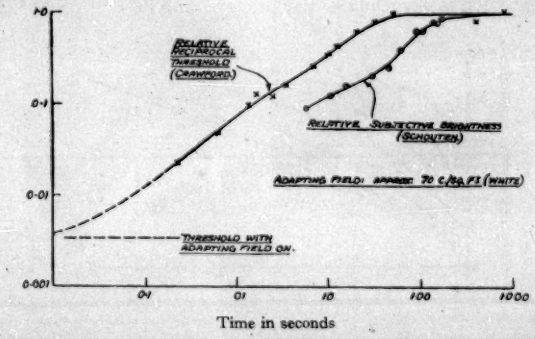

Figure 15. Comparison of the recovery curves for subjective brightness and for the reciprocal threshold after 5 min. exposure to an adapting field of 70 c./sq. ft. (From Schouten, 1939, and Crawford, 1937.)

Measurements of the subjective brightness of white stimuli give no information about the relative rates of recovery of the different cone mechanisms. Wright (1934) has shown that information of this kind can be obtained by the method of binocular colour matching. The test light seen by the recovering eye is colour-matched by a mixture of three spectral primaries forming the comparison light seen by the control eye. As recovery proceeds, the varying amounts of the primaries required determine three recovery curves. One of the first problems attacked by Wright was the determination of the visibility curves of the three cone mechanisms, or the fundamental response curves. The underlying assumption was the so-called law of coefficients. This says that if x, y, x, are the quantities of the fundamental primaries (i.e. hypothetical primaries each of which stimulates only one of the cone mechanisms) which match the test light before the application of the adapting light, then at a given time t after removing the adapting light the quantities required will be ax, by, cz, where the coefficients a, b, c are independent of the intensity and colour of the test light. They will, of course, depend on the characteristics of the adapting

light and the time *t*. By experimenting with various test lights and adapting lights, Wright was able to derive a set of fundamental response curves which seemed satisfactory, except for the fact that the green response curve dropped to negative values in the blue end of the spectrum. Although negative values are quite acceptable in colorimetry, they cannot easily be interpreted as a property of a cone mechanism. Recently Walters (1942) has shown that the coefficient law is not true in general, and has suggested that it becomes true only in the limiting case of a test stimulus of very low intensity. On this view, he has made a redetermination of the red and green response curves.

The coefficient law may fail because the test light itself is sufficiently bright to modify the recovery process, and this modification may occur to different extents for the three mechanisms. This would not necessarily entail any interaction between the three mechanisms. However, there is other evidence of true interaction between the mechanisms. A specially noteworthy effect is the phenomenon of the positive blue (Wright, 1937; Walters, 1942). If the retina is adapted with a strong red light, and a red test light is applied, then within the first few seconds of recovery the test-stimulus appears desaturated or, possibly, purplish, and positive blue has to be introduced into the comparison patch. It might be expected that the red adapting-light would fatigue the blue mechanism less than the red or green, but then the red test-stimulus would also stimulate the blue mechanism less. The evidence now accumulated seems to rule out any explanation of the positive blue on the basis of three independent mechanisms with constant spectral response curves. It should be noted that the effect cannot be attributed to a blue after-image superposed on, but otherwise independent of, the test-light, for it can occur under conditions where no blue after-image is visible in the absence of the test light.

Another type of interaction occurs if the rate of recovery of, say, the green mechanism depends on the instantaneous conditions or on the rates of recovery of the other two mechanisms. Such interaction might not of itself entail a breakdown in the coefficient law. That interaction of the kind in question does occur has been shown by Wright (1937), who found that the red and green recovery curves followed different courses, depending on whether, in the initial adaptation, the blue mechanism was, or was not, highly stimulated. It is not difficult to conceive how interaction of this type could be brought about if the recovery of the three mechanisms depended on some common and limited reservoir of photochemical material or on some common recuperative substance, such as oxygen carried by the blood.

On the whole it seems that the notion of three independent cone mechanisms with fixed response curves works fairly well, provided the retina has become adapted to the radiation falling on it. It is in the process of changing from one state of adaptation to another that the effects of interaction are chiefly exhibited.

One difficulty in interpreting many of the results so far obtained by the method of binocular colour matching is the fact that the various stimuli are expressed in terms of the instrument primaries. When the fundamental response curves are known with reasonable certainty, and all stimuli can be expressed in terms of them, it will be easier to comprehend what is happening in rather complicated experiments of this kind.

§ 5. FUNDAMENTAL RESPONSE CURVES

König believed that the principal colour-blinds—protanopes, deuteranopes, tritanopes—differed from the normal simply by the lack of one of the three response mechanisms, and his proposed fundamental response curves—the Grundempfindungen—were chosen largely to fit in with this idea. New measurements by Pitt (1944) of the colour-mixture curves, the visibility curve and the hue discrimination of protanopes and deuteranopes, and a reconsideration by him of König's observations on five tritanopes, lead him to the following conclusions. Protanopes lack the red mechanism, tritanopes the blue mechanism, while deuteranopes have in place of the red and green mechanisms a single mechanism whose response curve is a weighted mean of the normal red and green curves. His fundamental response curves based on this view and on some other evidence are shown in figure 16. As regards shape and position of the maxima,

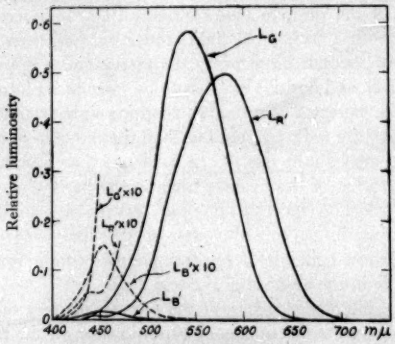

Figure 16. Fundamental response curves for an equi-energy spectrum derived in the main from a consideration of the properties of dichromatic vision. (From Pitt, 1944, reproduced by permission of the Council of the Royal Society.)

these curves are of the same general type as König's. Walters' red and green response curves from binocular matching are consistent with Pitt's, but they extend only down to about 520 mμ, as the blue curve was not determined. Measurements of the foveal threshold increment, referred to earlier, lead to the curves shown in figure 17. These are reciprocal threshold curves, and the relative ordinates of the different curves are not comparable with those of the Pitt curves. In shape and position the two sets, arrived at by very different methods, are in tolerably good agreement.

Helmholtz made a most interesting attempt to determine the response curves by applying the threshold condition in the form

$$1 = \left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2.$$

He applied this expression to König and Dieterici's measurements of the hue-discrimination sensitivity in the spectrum and found that to explain the experi-

mental results the response curves had to have the forms shown in figure 18. These are remarkably different from the König type, and, as Schrödinger pointed out in 1920, they lead to an impossible visibility curve. The modified threshold condition (3 *a*) shows promise of reconciling the correct visibility curve and hue-discrimination sensitivity with response curves of the König type.

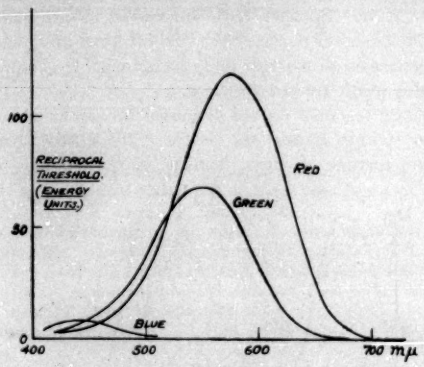

Figure 17. Reciprocal thresholds of the three cone mechanisms. (From Stiles, 1939.)

Of recent years a great many measurements have been made, both in this country by Wright and his co-workers (Wright and Pitt, 1935; Wright, 1941) and in America by MacAdam (1942), on the hue limen and the more general colour limen throughout the colour triangle. The results have been expressed

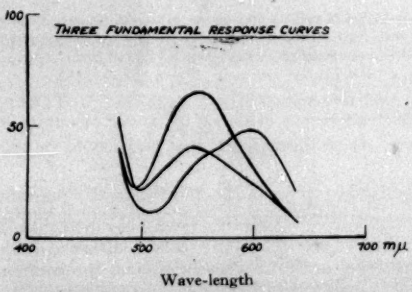

Figure 18. Fundamental response curves derived from the data of hue discrimination in the spectrum. (From Helmholtz, 1896.)

in terms of the C.I.E. trichromatic system and the so-called uniform-chromaticity system. Presumably if expressed in terms of the fundamental primaries they would assume a particularly significant form. This does not mean that when so expressed they would exhibit any very obvious or simple property. In

particular, they will not show that, in the triangle or in the rectangular system, just-distinguishable colours are always separated by a fixed distance. It now seems certain that no linear transformation of the C.I.E. co-ordinates would present the limen measurements in this simple form. The analysis of the data is complicated by the existence of considerable differences between the results of different observers. By the use of *non-linear* transformations of the C.I.E. co-ordinates, Moon and Spencer have succeeded in expressing the complete liminal data of one observer in the much desired form in which a fixed distance in the diagram separates all pairs of just distinguishable colours. The physical implications of this result are not yet clear.

The fundamental response curves proposed by Hecht in 1932 all have their maxima very near (within 10 mμ) the centre of the visible spectrum (555 mμ). They give a good picture of many features of trichromatic vision, but were

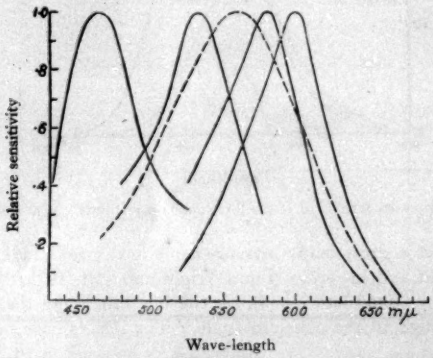

Figure 19. Sensitivity curves of various end-organs in the light-adapted retina of the frog, obtained by measuring the electrical response to light using a micro-electrode technique: the four continuous curves are classified as belonging to "modulator" elements, the broken curve as belonging to a "dominator" element. (From Granit, 1943.)

derived before the most recent evidence in favour of curves of the König type had been obtained. It seems unlikely that they could be reconciled with this evidence.

It would be gratifying to conclude this lecture by pointing to three photochemical substances, extracted from the retina and possessing spectral absorption curves agreeing with the fundamental response curves. This cannot be done, although recent progress in the photochemistry of the retina may make us not unhopeful. Granit's latest studies (1943) of the electrical response of animal retinae show that the latter contain mechanisms with the kind of spectral sensitivity curves we should require. The considerable variety of such curves obtained by Granit and his co-workers, a number of which are depicted in figure 19, may confront us in the future with the new problem of finding functions for more cone-sensitivity curves than the bare minimum of three required by psychophysical measurements of vision.

§ 6. ACKNOWLEDGEMENT

The delivery and publication of this lecture are by permission of the Director of the National Physical Laboratory.

REFERENCES

ABNEY, W. DE W. and WATSON, W., 1916. *Philos. Trans.* **216**, 91.
BARNES, A. B. and CZERNY, M., 1932. *Z. Phys.* **79**, 436.
BRUMBERG, E. and VAVILOR, S., 1933. *Bull. Aacd. Sci. U.R.S.S.* p. 919.
BUCKLEY, H., 1941. *Rep. Prog. Phys.* **8**, 318.
CRAIK, K. J. W., 1940. *Proc. Roy. Soc.* B, **128**, 232.
CRAWFORD, B. H., 1937. *Proc. Roy. Soc.* B, **123**, 69.
CRAWFORD, B. H., 1940. *Proc. Roy. Soc.* C.
DRESLER, A., 1940. *Das Licht*, **10**, 79.
GRANIT, R., 1943. *Nature, Lond.*, **151**, 11.
GUILD, J., 1932. " Discussion on Vision " (London : Phys. and Opt. Soc.), p. 14.
HECHT, S., 1932. " Discussion on Vision " (London : Phys. and Opt. Soc.) p. 126.
HECHT, S. and WILLIAMS, R. E., 1922. *J. Gen. Physiol.* **5**, 1.
HECHT, S., SHLAER, S. and PIRENNE, M. H., 1942. *J. Gen. Physiol.* **25**, 819.
HELMHOLTZ, H. VON, 1896. *Handb. d. Physiol. Optik* (2nd Edit.), **448**, 453.
KATZ, D., 1935. *The World of Colour* (London : Kegan, Paul and Co.).
LIVINGSTON, P. C., 1944. *Lancet* (Moyinham Lectures, Roy. Coll. Surgeons).
MACADAM, D. L., 1942. *J. Opt. Soc. Amer.* **32**, 247.
MEATHAM, A. R. and LAMBERT, G. E. V., 1942. Report to Ministry of Home Security (unpublished).
MOON, P. and SPENCER, D. E., 1943. *J. Opt. Soc. Amer.* **33**, 260.
PIRENNE, M. H., 1943. *Nature, Lond.*, **152**, 698.
PITT, F. H. G., 1944. *Proc. Roy. Soc.* B, **132**, 101.
PRESTON, J. S., 1939. *Proc. Phys. Soc.* **51**, 757.
SCHOUTEN, J. F., 1939. *J. Opt. Sc. Amer.* **29**, 168.
SCHRÖDINGER, E., 1920. *Ann. Phys., Lpz.*, **63**, 493.
STILES, W. S. 1932. " Discussion on Vision " (London : Phys. and Opt. Soc.), p. 326.
STILES, W. S., 1939. *Proc. Roy. Soc.* B, **127**, 64.
STILES, W. S., 1944. Unpublished work.
STILES, W. S. and SMITH, T., 1944. *Proc. Phys. Soc.* **56**, 251.
WALTERS, H. V., 1942. *Proc. Roy. Soc.* B, **131**, 27.
WALTERS, H. V. and WRIGHT, W. D., 1943. *Proc. Roy. Soc.* B, **131**, 340.
WRIGHT, W. D., 1934. *Proc. Roy. Soc.* B, **115**, 49.
WRIGHT, W. D., 1937. *Proc. Roy. Soc.* B, **122**, 220.
WRIGHT, W. D., 1941. *Proc. Phys. Soc.* **53**, 93.
WRIGHT, W. D. and PITT, F. H. G., 1935. *Proc. Phys. Soc.* **47**, 205.

DISCUSSION

Dr. K. J. W. CRAIK. While Hecht and Pirenne have certainly shown that, under ideal conditions, quantum fluctuations can reveal themselves in the measurement of the absolute visual threshold, I think one should not underestimate the part played by physiological fluctuations under most conditions. It may be possible to separate the two under certain circumstances. For instance, workers in this country and in America have found slow or rhythmic fluctuations of the absolute threshold having a mean period as long as 2 or 3 minutes. I think it would be possible to show that such a slow and sometimes regular periodicity is unlikely in the physical fluctuations of about 7 quanta. Again, referring to the suggestion that the differential brightness sensitivity of the eye at high brightnesses may be limited by the fluctuations in the number of quanta absorbed in the brighter and dimmer portions of the retinal image, one can take the case of exposing the eye to a bipartite photometric field at a brightness higher than that to which it has been adapted, and making a rapid measurement before the state of adaptation has readjusted itself. Such experiments by W. D. Wright and, later, by myself have shown that the

percentage brightness-difference threshold is no better, or slightly worse, under these conditions than for an eye adapted to the test brightness ; yet as there should be a higher concentration of photochemical substance in the first case, more quanta should be absorbed and there should be less physical fluctuation in the difference between the number absorbed in the brighter and dimmer halves of the image.

Mr. J. GUILD. In this survey of visual problems Dr. Stiles has succeeded in showing not only how very considerable is the progress which has been made in recent years in the field of visual research, but also how vast this field is and how much of it still remains to be explored. He has touched on many problems, each of which could profitably be the subject of a long discussion, but I shall confine my comments to one topic.

I wish to draw attention to the increasingly important part that statistical techniques are playing in these investigations. We are becoming increasingly familiar with the S-shaped curve of some probability integral representing the relation between stimulus values and the chance of seeing something or other under certain conditions. Most of the older methods of determining liminal quantities implied a concept of a limen as a minimal stimulus (or stimulus difference) for sensory detection. The concept did not include fluctuations : and those which were, in fact, always encountered in liminal experiments were regarded as having a nuisance value only, inasmuch as they made it more difficult to determine the true value of the limen. They were considered in the same light as experimental errors arising from inadequate precision in some measuring apparatus, to be eliminated by taking the average of a sufficient number of observations. But the fluctuations of a limen should not be regarded as mere sources of error in determining some unique quantity—a threshold, or a just noticeable difference, for example. They are as much a characteristic of the phenomena investigated as any mean value or other single value derivable from the experiments, and a study of the statistical laws which they follow is necessary to the solution of many problems of practical or theoretical interest. The fluctuations are large, the effective value in any given instance being not infrequently as low as 25 or as high as 175 per cent of the average value, while still more extreme values are occasionally operative. In these circumstances the mean value has little special significance, and the classical concept of the limen, in terms of which problems of sensory discrimination have been couched since the time of Weber and Fechner, seems to be of very limited utility.

I suggest that instead of regarding sensory discrimination as determined by *a* limen appropriate to the sensory organ under the prevailing conditions, we should regard it as determined by the statistical frequency-distribution of the values which the effective limen may have at different instants. This distribution does not appear to be quite symmetrical above and below the mean, but is in most cases sufficiently nearly so over the greater part of the range to be adequately approximated, for most practical purposes, by a normal (Gaussian) distribution. For these purposes the limen is adequately described by its mean value and standard deviation. This widening of the concept of the limen has in itself been of great assistance in the solution of some important visual problems ; but so long as the statistical approach is purely actuarial, it merely facilitates prediction of visual behaviour without throwing much light on the underlying causes.

The work of Hecht and his colleagues, to which Dr. Stiles has referred, marks a break-away from the purely actuarial approach : they regard the statistical distribution not merely as descriptive of the observations but also, in its form, as related to the causes which determine them. This principle is not new—I am told it was employed many years ago in connection with some photographic problems, and there have no doubt been other applications—but it is very rarely utilized in connection with physical investigations, and, as far as I know, this was its first application to a psychophysical problem. Hecht inferred from the fact that observations of flash thresholds could be fitted by one particular member of the class of Poisson distributions, that a definite number of elementary events—six or seven—were involved in seeing a flash. The theory he builds on this has been indicated by Dr. Stiles. It is not primarily on his results that I wish to comment, but on the far-reaching consequences of his method, which opens up a wide field for future investigations. But because of its importance it behoves us to beware of its limitations. Inferences respecting the causes of an observed variation derived from the form of the statistical distribution are only justified to the extent that the observations serve to identify

the particular distribution to the exclusion of other forms which would imply quite different causal systems. No finite number of observations can determine a distribution exactly : there is always some uncertainty as to the functional relation which best fits them, and, whatever one we choose, residual discrepancies are to be expected. If we are merely interested in getting a good formal approximation to the actual observations, our work is finished when we have found a functional relation which is at least as good a fit as any other we might choose ; but if we intend to attach significance to the relation having one form rather than another it is not sufficient to show that the actual observations are fitted better by form A than by form B : they might be so, as the result of chance errors, even if the true form of the distribution of which they are a random sample were B. It is necessary to show that the probability of the actual observations having occurred on any hypothesis of the form B is sufficiently low, either in itself or in comparison with the probability of the same observations on hypothesis A, to afford reasonable evidence that form A is actually the true form of the distribution. Hecht's results are not accurately fitted by a Poisson curve; but even if they were it would still be pertinent to enquire whether this was significant or merely the result of a chance throw-up of values from a distribution of some other form—say the normal form—with very different causal implications. It is thus necessary to find also the best-fitting normal distribution and employ some of the well established *goodness-of-fit* criteria to find the probability that discrepancies as great as those recorded might arise from the process of random sampling. Unless this probability is fairly low the observations, however well they may be fitted by a Poisson curve, do not justify the inference that the Poisson form is specially appropriate. Without more details of the observations than is given in the published accounts of the work it is not practicable to make a reliable test of this kind, and without it we do not know what weight to attach to Hecht's results when regarding them as evidence that the fluctuations of the flash threshold are really determined by Poisson's law of small numbers rather than by the normal law of variation, which tells us very little about the number of events involved. Hecht states that no special statistical tests are necessary to determine which curve fits the data, the appropriate curve being easily selected by the simplest visual comparison. This is only true on the assumption that a Poisson distribution is necessarily involved. Granting this, the selection of the appropriate Poisson curve by inspection may present no difficulty. But to establish with high probability that the parent distribution, of which the observations are only a random sample, is actually of this type requires more elaborate tests.

There is, however, other evidence that the Poisson law is appropriate to threshold and liminal fluctuations—at any rate for visual perception. Prior to the publication of Hecht's work I had carried out an investigation, not available for publication, in the course of which it was necessary to make a statistical analysis of the brightness limen under certain conditions. Some 3000 observations were involved, and in half of these the method of experiment was such that every observation determined a point on the S curve of the probability integral. The curve was, therefore, determined with considerable precision throughout the range. Except at the foot and shoulder, a very close fit was afforded by the normal law of distribution, which served perfectly for the practical objects for which the work was done : but after reading Hecht's paper and perceiving the great importance of the principle he had employed I re-examined these results and found that a Poisson distribution gave a better fit at the extremities of the S curve. The question of principle still remains : could the departures from a normal distribution be reasonably attributed to chance ? Application of the χ^2 test showed that the probability of the observations being a random selection from a normal distribution was certainly less than 0·001, whereas their probability on the hypothesis of a Poisson distribution was 0·95. It is therefore reasonably certain that the normal law cannot account for the observations, and that the Poisson law is operative. I may not give details of these experiments, beyond indicating that they apply to foveal vision and photopic conditions. Under the conditions which prevailed, the brightness limen had a mean value of 1·5 per cent for monocular vision and 1·1 per cent for binocular vision, with a standard deviation which in each case was 24 per cent of the mean value. Any difference there may be between binocular and monocular vision in respect of the standard deviation of the limen, when this is expressed as a fraction of the mean value, is too small to be detected without a very much greater number of observations. The same Poisson distribution fits both monocular and binocular

data and the minimum number of elementary events involved is 17, as compared with six or seven in the very different conditions of Hecht's experiments. It seems practically certain, therefore, that not only under threshold conditions, but also at high brightness levels, discrimination is determined by the collective effect of some definite small number of events. Clearly, however, the results so far obtained must be supplemented by many others to determine how this number depends on field size, field brightness, and other experimental conditions before we can hope to proceed very far with interpretation; but enough has been established to show the importance and interest of the line of investigation which has been opened up by the researches of Hecht and his co-workers.

Dr. W. D. Wright. Without disputing the elegance of the method by which Hecht has attempted to deduce the number of quanta required to stimulate the eye at the absolute threshold of vision from a consideration of the scatter of the experimental observations, I do not believe he has allowed correctly for the physiological variations which may occur in the visual mechanism. His reference to these factors suggests that they will vary as a continuous function, whereas in fact the nerve currents have the same quantum character as has light radiation. Thus it would seem to me that the same scatter in the results might be anticipated if, say, six nerve impulses were, on the average, necessary to arouse a threshold sensation, as if six light quanta were on the average necessary to cause some given photochemical decomposition. Admittedly, from independent observations, it might be found that six quanta were required, and hence the whole of the scatter of the observations mgiht be attributed to quantum variations, but that has not been Hecht's line of argument.

I was very interested to note the agreement between Dr. Stiles's theoretical luminosity curves in the mixed scotopic-photopic range with the experimental curves as measured by Walters and myself. When we attempted to deduce the mixed curve from the separate photopic and scotopic curves, we came to the conclusion that this was almost impossible without knowing more about the mechanism through which the cones take over from the rods as the intensity is raised. I shall, therefore, look forward to studying in detail how Dr. Stiles has succeeded in short-circuiting information about this process.

Lieut. M. H. Pirenne. The consideration of physical fluctuations and probabilities in visual research seems to open up promising fields. Caution, however, must accompany enthusiasm. For it is not possible, for example, to deduce directly the number of quanta involved in a process from the shape of a frequency curve when the latter is a combined curve due to several independent receptors. We must know the details of the physiological mechanisms in order to do so. As a basis for this, it is to be hoped that the anatomists will soon give us quantitative information about the connections between receptors and nerve fibres over the whole of the retina—a difficult problem, but well worth the trouble.

In connection with the differential sensitivity at high brightness, it occurred to me long ago that if a receptor absorbs during its summation time an average number N of quanta, the smallest variation in N detectable by the organism will be of the order of twice the standard deviation of N, or $2\sqrt{N}$. (The similarity to the absolute limit set by Brownian movement to the accuracy of galvanometers is obvious.) Our present information is not complete enough to allow us to calculate exactly whether this limit is ever actually reached. But the following rough calculation, which should be taken only as an indication, shows that this is not impossible. König and Brodhum (*Sitzb. Akad. Wiss. Berlin*, 1888, p. 917, reprinted in König, A., *Ges. Abh. z. Physiologischen Optik*, p. 116 (Leipzig, 1903)) found that for $\lambda = 0{\cdot}670\mu$ the curve of $\Delta I/I$ plotted against I flattens out at a brightness I equal to about 17,000 times the absolute threshold, $\Delta I/I$ reaching a value of 0·017 to 0·016. If we assume that the number of quanta absorbed by one receptor during its summation time at the absolute threshold is of the order of 10, and that the absorption by the photosensitive substance at the higher brightnesses is reduced to 1/10 of its original value, the number N is 17,000 and $2\sqrt{N}/N = 0{\cdot}015$, in agreement with the observed $\Delta I/I$ values. Steinhardt (*J. Gen. Physiol.* **20,** 1936, 197), observed with very large fields a persistent elevation of the measured $\Delta I/I$ over the theoretical (photochemical) curve at high intensities. A possible explanation of these results would be that the absolute physical limit was actually reached in Steinhardt's experiments.

As regards the remarks of Dr. Wright and Dr. Craik on the biological variations of the threshold, I should say, first, that in the investigation by Hecht, Shlaer and Pirenne the emphasis laid on placing the observer under very stable conditions clearly implies that we recognized that factors such as fatigue very probably determine biological variations of sensitivity. But the physical fluctuations will always remain present, so that only statistical analysis can decide whether long-range changes in threshold, for instance, mean a biological change of sensitivity or not.

Secondly, the whole point of our investigation is that the quantum fluctuations which must be expected are so large that there is little room left for biological variations. Theoretical arguments for the existence of large biological variations therefore do not seem able to carry much weight at the present stage. Moreover, it is not likely that, for instance, the inaccuracy of fixation, mentioned by Dr. Wright, can cause large variations of sensitivity under the conditions we used, chiefly because this inaccuracy is of the order of 2′, while the test field has a diameter of 10′, covering about 500 rods on the retina.

Granit (*Acta Physiol. Scand.* **1**, 1941, 370), in recording electrically the impulses in single fibres of the retina, found at the threshold a "fluctuation of excitability" to which he did not ascribe a definite origin. If the analysis of the records showed this variability to be purely random under constant conditions, it might be suggested that it is actually due to quantum fluctuations in the absorption of light by the receptors.

Author's reply. The fresh experimental evidence on the S curve brought forward by Mr. Guild opens a new phase of the problem. If the intensity for 50 per cent seeing-chance and the approximate slope of the S curve at this intensity are regarded, respectively, as the first- and second-order quantities associated with the threshold, then Mr. Guild is concerned with third- or higher-order quantities which determine the *precise* shape of the curve. Hecht, Shlaer and Pirenne's contribution is at the level of the second-order quantity; they have tried to show, in effect, that the observed slope of the S curve has approximately the value appropriate to quantum fluctuations of the small number of quanta which must be absorbed by the visual purple to produce a vision response, *this number being independently estimated*. Dr. Wright questions whether this is H., S. and P.'s line of argument, but I can read no other into their published papers. If the observed slope had been found much less than the estimated slope, this would have indicated a considerable biological fluctuation, which might also have had a "quantum" origin, associated possibly with individual nerve pulses, as Dr. Wright suggests. In that case, H., S. and P.'s experiments would not have warranted the conclusion that the response depended on a fairly small number of *discrete* events, for, as Mr. Guild points out, their S curves are not determined accurately enough to say whether a Gaussian or Poisson law would fit better. For Mr. Guild's curve, based on many more observations, such a distinction is possible and the Poisson law is found to be better. But the effective number of radiational quanta involved in the visual act in Mr. Guild's experiment cannot, at present, be independently estimated. It is not possible to decide, therefore, whether the 17 discrete events to which he refers are absorptions of radiational quanta, activations of individual nerve fibres, or some composition of events of these two categories.

Dr. Craik refers to suspected short-period rhythmic changes in the threshold, and most investigators have noted, on occasion, drift variations, when the observations are spread over long periods, as well as day-to-day changes. In the very large number of observations necessary to determine the precise shape of the S curve, any effects of this kind would be lumped in with those fluctuations which continue to occur in however short a time a series of observations is compressed. The separation of fluctuations of these two kinds is a difficult problem which must be faced when interpreting the S curve derived from large sets of observations.

Dr. Craik and Lieut. Pirenne have discussed the relation between quantum fluctuations and the *actual value* of the differential brightness threshold at high brightnesses. While I cannot adequately comment here on their remarks, I may mention that in my reference to higher brightness levels in the lecture, I had in mind the simpler situation which arises when the eye views, and is adapted to, a high brightness and the threshold, foveal or parafoveal, is again determined for a flashing point-source. The adapting brightness may so reduce the concentration of photochemical substance in all receptors that the

fraction of the light flash which is absorbed is reduced to a number of quanta sufficiently small to account for the observed scatter of the measured threshold. This picture shows merely how quantum fluctuations may still account for the scatter of the threshold at high brightness. More specific assumptions are needed in drawing any deductions about the actual value of the threshold.

In the derivation of the luminosity curves in the mixed scotopic-photopic range, the "short-circuiting" to which Dr. Wright refers consists, in the main, of making use of the fact that in threshold curves like those of figure 6 the rod and cone component curves intersect fairly sharply.

DISCUSSION

on paper by J. C. JAEGER, *Proc. Phys. Soc.* **56**, 197 (1944), entitled "Note on a problem in radial flow".

Mr. H. H. MACEY. I am indebted to Dr. Jaeger for the interest which has led him so carefully to examine this idea of mine concerning the correspondence between one- and two-dimensional flow curves, and, in particular, for the large amount of labour which must have been involved in the different calculations.

The object of utilizing this correspondence is to obtain numerical values for the distribution of moisture-content or temperature at small times in the cylinder whilst avoiding the use of (13), which is arithmetically laborious and confusing in changes of sign. It is, in the practical case, required to know the value of the corresponding time t_1 of the bar from experimental knowledge of the time t for the cylinder. I should like to point out that if more accuracy is desired than can be obtained from the curves given by Dr. Jaeger it is not necessary to use (30) for this purpose. For small values of t_1, and at the surface, $r=a$, (24) reduces to $v_1=V-\dfrac{1{\cdot}1284Q(\kappa t_1)^{\frac{1}{2}}}{K}$, since $2X_1(0)=2/\sqrt{\pi}=1{\cdot}128379$. Similarly at the surface in (13) the Bessel functions disappear, and the summation is straightforward. Equating v_1 and v at the surface gives $(\kappa t_1)^{\frac{1}{2}}$ for direct use in (24). As an example of the accuracy involved, an experimental value of κt was 1·608. The corresponding value of κt_1 calculated in this way is 1·724. Recalculating κt from (28) gives 1·6117, which is almost within the accuracy of the four-figure tables used. If t or t_1 be so large that the second term in (24) at the surface is not zero to four-decimal-place accuracy, the simple parabolic solution may be used.

CORRIGENDA

(1) In *Proc. Phys. Soc.* **56**, 212 (Part 3, **May** 1944), line 3, for "SbSi ($?^{1}$)" read "SbBi ($?\,^{1}\Sigma$)".

(2) In *Proc. Phys. Soc.* **56**, 239 (Part 4, July 1944), Dr. Llewellyn Jones was described as of "Bristol University". This should be "University College of Swansea".